UNIT 2

Powers and Exponent Laws

What You'll Learn

- Use powers to show repeated multiplication.
- Evaluate powers with exponent 0.
- Write numbers using powers of 10.
- Use the order of operations with exponents.
- Use the exponent laws to simplify and evaluate expressions.

Why It's Important

Powers are used by

- lab technicians, when they interpret a patient's test results
- reporters, when they write large numbers in a news story

Key Words

integer
opposite
positive
negative
factor
power
base
exponent
squared
cubed
standard form
product
quotient

2.1 Skill Builder

Multiplying Integers

When multiplying 2 integers, look at the sign of each integer:

- When the integers have the same sign, their product is positive.
- When the integers have different signs, their product is negative.

×	(−)	(+)
(−)	(+)	(−)
(+)	(−)	(+)

$6 \times (-3)$ These 2 integers have different signs, so their product is negative.

$6 \times (-3) = -18$

$(-10) \times (-2)$ These 2 integers have the same sign, so their product is positive.

$(-10) \times (-2) = 20$

When an integer is positive, we do not have to write the + sign in front.

Check

1. Will the product be positive or negative?

a) 7×4 **Positive**

b) $3 \times (-6)$ **Negative**

c) $(-9) \times 10$ **Negative**

d) $(-5) \times (-9)$ **Positive**

2. Multiply.

a) $7 \times 4 =$ **28**

b) $3 \times (-6) =$ **−18**

c) $(-9) \times 10 =$ **−90**

d) $(-5) \times (-9) =$ **45**

e) $(-3) \times (-5) =$ **15**

f) $2 \times (-5) =$ **−10**

g) $(-8) \times 2 =$ **−16**

h) $(-4) \times 3 =$ **−12**

Multiplying More than 2 Integers

We can multiply more than 2 integers.
Multiply pairs of integers, from left to right.

$$\underbrace{(-1) \times (-2)} \times (-3)$$
$$= 2 \times (-3)$$
$$= -6$$

The product of 3 negative factors is negative.

$$\underbrace{(-1) \times (-2)} \times (-3) \times (-4)$$
$$= \underbrace{2 \times (-3)} \times (-4)$$
$$= (-6) \times (-4)$$
$$= 24$$

The product of 4 negative factors is positive.

Multiplying Integers
When the number of negative factors is *even*, the product is positive.
When the number of negative factors is *odd*, the product is negative.

We can show products of integers in different ways:
$(-2) \times (-2) \times 3 \times (-2)$ is the same as $(-2)(-2)(3)(-2)$.

So, $(-2) \times (-2) \times 3 \times (-2) = (-2)(-2)(3)(-2)$
$= -24$

Check

1. Multiply.

a) $(-3) \times (-2) \times (-1) \times 1$ **−6**

b) $(-2)(-1)(-2)(-2)(2)$ **16**

c) $(-2)(-2)(-1)(-2)(-2)$ **−16**

d) $3 \times 3 \times 2$ **18**

Is the answer positive or negative? How can you tell?

TEACHER NOTE
For related review, see *Math Makes Sense 8*, Sections 2.1 and 2.2.

2.1 What Is a Power?

FOCUS **Show repeated multiplication as a power.**

TEACHER NOTE
Investigate on Student Text page 52 is well suited to hands-on and visual learners. Consider using mixed-ability groupings so that all students can participate.

We can use powers to show repeated multiplication.

$$\underbrace{2 \times 2 \times 2 \times 2 \times 2}_{\text{Repeated multiplication, 5 factors of 2}} = \underbrace{2^5}_{\text{Power}}$$

Repeated multiplication
5 **factors** of 2

Power

2 is the ***base.***
5 is the ***exponent.***
2^5 is a ***power.***

We read 2^5 as "2 to the 5th."
Here are some other powers of 2.

Repeated Multiplication	Power	Read as…
$\underbrace{2}_{\text{1 factor of 2}}$	2^1	2 to the 1st
$\underbrace{2 \times 2}_{\text{2 factors of 2}}$	2^2	2 to the 2nd, or 2 squared
$\underbrace{2 \times 2 \times 2}_{\text{3 factors of 2}}$	2^3	2 to the 3rd, or 2 cubed
$\underbrace{2 \times 2 \times 2 \times 2}_{\text{4 factors of 2}}$	2^4	2 to the 4th

In each case, the exponent in the power is equal to the number of factors in the repeated multiplication.

Example 1 Writing Powers

Write as a power.

a) $4 \times 4 \times 4 \times 4 \times 4 \times 4$

b) 3

Solution

a) The base is 4.

$$\underbrace{4 \times 4 \times 4 \times 4 \times 4 \times 4}_{\text{6 factors of 4}} = 4^6$$

So, $4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^6$

b) The base is 3.

$$\underbrace{3}_{\text{1 factor of 3}}$$

So, $3 = 3^1$

Check

1. Write as a power.

a) $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^{6}$

b) $5 \times 5 \times 5 \times 5 = 5^{4}$

c) $(-10)(-10)(-10) = \mathbf{(-10)^3}$

d) $4 \times 4 = \mathbf{4^2}$

e) $(-7)(-7)(-7)(-7)(-7)(-7)(-7)(-7) = \mathbf{(-7)^8}$

2. Complete the table.

	Repeated Multiplication	Power	Read as…
a)	$8 \times 8 \times 8 \times 8$	$\mathbf{8^4}$	8 to the 4th
b)	7×7	$\mathbf{7^2}$	**7 squared**
c)	$3 \times 3 \times 3 \times 3 \times 3 \times 3$	$\mathbf{3^6}$	3 to the 6th
d)	$2 \times 2 \times 2$	$\mathbf{2^3}$	**2 cubed**

Power	Repeated Multiplication	Standard Form
2^5	$2 \times 2 \times 2 \times 2 \times 2$	32

Example 2 Evaluating Powers

Write as repeated multiplication and in standard form.

a) 2^4

b) 5^3

Solution

a) $2^4 = 2 \times 2 \times 2 \times 2$ — As repeated multiplication
$= 16$ — Standard form

b) $5^3 = 5 \times 5 \times 5$ — As repeated multiplication
$= 125$ — Standard form

Check

1. Complete the table.

Power	Repeated Multiplication	Standard Form
2^3	$2 \times 2 \times 2$	**8**
6^2	**6×6**	36
3^4	**$3 \times 3 \times 3 \times 3$**	**81**
10^4	**$10 \times 10 \times 10 \times 10$**	**10 000**
8 squared	**8×8**	**64**
7 cubed	**$7 \times 7 \times 7$**	**343**

TEACHER NOTE

A common student error is to interpret 2^3 as 2×3 or 6. Assist students by relating the power to the concrete model of a cube. Highlight that $2^3 = 2 \times 2 \times 2 = 8$.

To evaluate a power that contains negative integers, identify the base of the power. Then, apply the rules for multiplying integers.

Example 3 Evaluating Expressions Involving Negative Signs

Identify the base, then evaluate each power.

a) $(-5)^4$

b) -5^4

Solution

a) $(-5)^4$

The brackets tell us that the base of this power is (-5).

$$(-5)^4 = (-5) \times (-5) \times (-5) \times (-5)$$
$$= 625$$

There is an even number of negative integers, so the product is positive.

b) -5^4

There are no brackets. So, the base of this power is 5. The negative sign applies to the whole expression.

$$-5^4 = -(5 \times 5 \times 5 \times 5)$$
$$= -625$$

Check

1. Identify the base of each power, then evaluate.

a) $(-1)^3$
The base is **−1**.
$(-1)^3 = \mathbf{(-1)(-1)(-1)}$
$= \mathbf{-1}$

b) -10^3
The base is **10**.
$-10^3 = \mathbf{-(10 \times 10 \times 10)}$
$= \mathbf{-1000}$

c) $(-7)^2$
The base is **−7**.
$(-7)^2 = \mathbf{(-7)(-7)}$
$= \mathbf{49}$

d) $-(-5)^4$

The first negative sign applies to the whole expression.

The base is **−5**.
$-(-5)^4 = \mathbf{-(-5)(-5)(-5)(-5)}$
$= \mathbf{-625}$

Practice

1. Write as a power.

a) $\underbrace{8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8}_{\text{7 factors of 8}}$

The base is 8. There are **7** equal factors, so the exponent is **7**.

$8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 = 8^{\mathbf{7}}$

b) $\underbrace{10 \times 10 \times 10 \times 10 \times 10}_{\text{5 factors of 10}}$

The base is **10**. There are **5** equal factors, so the exponent is **5**.

So, $10 \times 10 \times 10 \times 10 \times 10 = \mathbf{10^5}$

c) $\underbrace{(-2)(-2)(-2)}_{\text{3 factors of } \mathbf{-2}}$

The base is **−2**. There are **3** equal factors, so the exponent is **3**.

So, $(-2)(-2)(-2) = \mathbf{(-2)^3}$

d) $(-13)(-13)(-13)(-13)(-13)(-13)$
6 factors of **−13**

The base is **−13**. There are **6** equal factors, so the exponent is **6**.

So, $(-13)(-13)(-13)(-13)(-13)(-13) = \mathbf{(-13)^6}$

2. Write each expression as a power.

a) $9 \times 9 \times 9 \times 9 = \mathbf{9^4}$

b) $(5)(5)(5)(5)(5)(5) = 5^{\mathbf{6}}$

c) $11 \times 11 = \mathbf{11^2}$

d) $(-12)(-12)(-12)(-12)(-12) = \mathbf{(-12)^5}$

3. Write each power as repeated multiplication.

a) $3^2 = \underline{\mathbf{3 \times 3}}$

b) $3^4 = \underline{\mathbf{3 \times 3 \times 3 \times 3}}$

c) $2^7 = \underline{\mathbf{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}}$

d) $10^8 = \underline{\mathbf{10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10}}$

Identify the base first.

4. State whether the answer will be positive or negative.

a) $(-3)^2$ **Positive**

b) 6^3 **Positive**

c) $(-10)^3$ **Negative**

d) -4^3 **Negative**

5. Write each power as repeated multiplication and in standard form.

Predict. Will the answer be positive or negative?

a) $(-3)^2 = \mathbf{(-3)(-3)}$
$= \mathbf{9}$

b) $6^3 = \mathbf{6 \times 6 \times 6}$
$= \mathbf{216}$

c) $(-10)^3 = \mathbf{(-10)(-10)(-10)}$
$= \mathbf{-1000}$

d) $-4^3 = \mathbf{-(4 \times 4 \times 4)}$
$= \mathbf{-64}$

6. Write each product as a power and in standard form.

a) $(-3)(-3)(-3) = \mathbf{(-3)^3}$
$= \mathbf{-27}$

b) $(-8)(-8) = \mathbf{(-8)^2}$
$= \mathbf{64}$

c) $-(8 \times 8 \times 8) = \mathbf{-8^3}$
$= \mathbf{-512}$

d) $-(-1)(-1)(-1)(-1)(-1)(-1)(-1) = \mathbf{-(-1)^7}$
$= \mathbf{1}$

7. Identify any errors and correct them.

a) $4^3 = 12$ — $\mathbf{4^3 = (4)(4)(4)}$
$\mathbf{= 64}$

b) $(-2)^9$ is negative. — **$(-2)^9$ is negative, because there is an odd number of negative factors.**

c) $(-9)^2$ is negative. — **$(-9)^2$ is positive, because there is an even number of negative factors.**

d) $3^2 = 2^3$ — **3^2 is not equal to 2^3, because $3^2 = (3)(3) = 9$, and $2^3 = (2)(2)(2) = 8$**

e) $(-10)^2 = 100$ — $\mathbf{(-10)^2 = (-10)(-10)}$
$\mathbf{= 100}$

TEACHER NOTE

Next Steps: Direct students to questions 7, 8, 9, 12, 13, and 14 on pages 55 and 56 of the Student Text.

2.2 Skill Builder

Patterns and Relationships in Tables

Look at the patterns in this table.

Input	Output
1	2
2	4
3	6
4	8
5	10

The input starts at 1 and increases by 1 each time.

The output starts at 2 and increases by 2 each time.

The input and output are also related.
Double the input to get the output.

Check

1. a) Describe the patterns in the table.

b) What is the input in the last row?
What is the output in the last row?

Input	Output
1	5
2	10
3	15
4	20
5	25

a) The input starts at 1, and increases by 1 each time.
The output starts at 5, and increases by 5 each time.
You can also multiply the input by 5 to get the output.

b) The input in the last row is 4 + 1 = 5.
The output in the last row is 20 + 5 = 25.

2. a) Describe the patterns in the table.

b) Extend the table 3 more rows.

Input	Output
10	100
9	90
8	80
7	70
6	60

a) The input starts at 10, and decreases by __1__ each time.
The output starts at 100, and decreases by __10__ each time.
You can also multiply the input by __10__ to get the output.

b) To extend the table 3 more rows, continue to decrease the input by __1__ each time.
Decrease the output by __10__ each time.

Input	Output
5	**50**
4	**40**
3	**30**

Writing Numbers in Expanded Form

8000 is 8 thousands, or 8×1000
600 is 6 hundreds, or 6×100
50 is 5 tens, or 5×10

Read it aloud.

Check

1. Write each number in expanded form.

a) 7000 **7 × 1000**

b) 900 **9 × 100**

c) 400 **4 × 100**

d) 30 **3 × 10**

TEACHER NOTE
For related review, see *Math Makes Sense 7*, Section 1.5.

2.2 Powers of Ten and the Zero Exponent

FOCUS **Explore patterns and powers of 10 to develop a meaning for the exponent 0.**

This table shows decreasing powers of 3.

Power	Repeated Multiplication	Standard Form
3^5	$3 \times 3 \times 3 \times 3 \times 3$	243
3^4	$3 \times 3 \times 3 \times 3$	81
3^3	$3 \times 3 \times 3$	27
3^2	3×3	9
3^1	3	3

$\div 3$

$\div 3$

$\div 3$

$\div 3$

Look for patterns in the columns.

The exponent decreases by 1 each time.

Divide by 3 each time.

The patterns suggest $3^0 = 1$ because $3 \div 3 = 1$.

We can make a similar table for the powers of any integer base except 0.

The Zero Exponent

A power with exponent 0 is equal to 1.

The base of the power can be any integer except 0.

Example 1 Powers with Exponent Zero

Evaluate each expression.

a) 6^0

b) $(-5)^0$

Solution

A power with exponent 0 is equal to 1.

a) $6^0 = 1$

b) $(-5)^0 = 1$

The zero exponent applies to the number in the brackets.

Check

1. Evaluate each expression.

a) $8^0 =$ __**1**__

b) $-4^0 =$ __**−1**__

c) $4^0 =$ __**1**__

d) $(-10)^0 =$ __**1**__

If there are no brackets, the zero exponent applies only to the base.

Example 2 Powers of Ten

Write as a power of 10.

a) 10 000 **b)** 1000 **c)** 100 **d)** 10 **e)** 1

Solution

a) $10\,000 = 10 \times 10 \times 10 \times 10$
$= 10^4$

b) $1000 = 10 \times 10 \times 10$
$= 10^3$

c) $100 = 10 \times 10$
$= 10^2$

d) $10 = 10^1$

e) $1 = 10^0$

Notice that the exponent is equal to the number of zeros.

Check

1. a) $5^1 =$ **5**

b) $(-7)^1 =$ **−7**

c) $10^1 =$ **10**

d) $10^0 =$ **1**

Practice

1. a) Complete the table below.

Power	Repeated Multiplication	Standard Form
5^4	$5 \times 5 \times 5 \times 5$	625
5^3	$5 \times 5 \times 5$	**125**
5^2	**5 × 5**	**25**
5^1	**5**	**5**

b) What is the value of 5^1? **5**

c) Use the table. What is the value of 5^0? **1**

2. Evaluate each power.

a) $2^0 = \underline{1}$ b) $9^0 = \underline{1}$

c) $(-2)^0 = \underline{1}$ d) $-2^0 = \underline{-1}$

e) $10^1 = \underline{10}$ f) $(-8)^1 = \underline{-8}$

If there are no brackets, the exponent applies only to the base.

3. Write each number as a power of 10.

a) $10\,000 = 10^{4}$ b) $1\,000\,000 = 10^{6}$

c) Ten million = $\underline{10^7}$ d) One = $\underline{10^0}$

e) $1\,000\,000\,000 = \underline{10^9}$ f) $10 = \underline{10^1}$

4. Evaluate each power of 10.

a) $-10^6 = \underline{-1\,000\,000}$ b) $-10^0 = \underline{-1}$

c) $-10^8 = \underline{-100\,000\,000}$ d) $-10^1 = \underline{-10}$

5. One trillion is written as 1 000 000 000 000.
Write each number as a power of 10.

a) One trillion = 1 000 000 000 000 = $\underline{10^{12}}$

b) Ten trillion = $10 \times \underline{1\,000\,000\,000\,000} = \underline{10^{13}}$

c) One hundred trillion = $\underline{100 \times 1\,000\,000\,000\,000} = \underline{10^{14}}$

6. Write each number in standard form.

a) $5 \times 10^4 = 5 \times 10\,000$
$= \underline{50\,000}$

b) $(4 \times 10^2) + (3 \times 10^1) + (7 \times 10^0) = (4 \times 100) + \underline{(3 \times 10) + (7 \times 1)}$
$= \underline{400 + 30 + 7}$
$= \underline{437}$

c) $(2 \times 10^3) + (6 \times 10^2) + (4 \times 10^1) + (9 \times 10^0)$
$= \underline{(2 \times 1000) + (6 \times 100) + (4 \times 10) + (9 \times 1)}$
$= \underline{2000 + 600 + 40 + 9}$
$= \underline{2649}$

d) $(7 \times 10^3) + (8 \times 10^0) = \underline{(7 \times 1000) + (8 \times 1)}$
$= \underline{7000 + 8}$
$= \underline{7008}$

TEACHER NOTE
Direct students to questions 4, 5, 6, 8, 9, and 10 on page 61 of the Student Text.

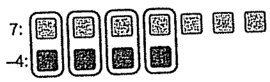
For students experiencing success, introduce Example 3 on page 60 of the Student Text, and assign Practice questions 11 and 14.

2.3 Skill Builder

Adding Integers

To add a positive integer and a negative integer: 7 + (–4)

- Model each integer with tiles.
- Circle zero pairs.



There are 4 zero pairs.
There are 3 tiles left.
They model 3.
So, 7 + (−4) = 3

Each pair of 1 tile and 1 tile makes a zero pair. The pair models 0.

To add 2 negative integers: (−4) + (−2)

- Model each integer with tiles.
- Combine the tiles.

There are 6 tiles.
They model −6.
So, (−4) + (−2) = –6

TEACHER NOTE
Encourage students to move to symbolic form when they are able.

Check

1. Add.

a) (−3) + (−4) = **−7**

b) 6 + (−2) = **4**

c) (−5) + 2 = **−3**

d) (−4) + (−4) = **−8**

2. a) Kerry borrows \$5. Then she borrows another \$5.
Add to show what Kerry owes.
(−5) + (−5) = **−10**

Kerry owes \$**10**.

When an amount of money is negative, it is owed.

b) The temperature was 8°C. It fell 10°C.
Add to show the new temperature.
8 + (**−10**) = **−2**

The new temperature is **−2°** C.

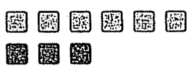
Subtracting Integers

To subtract 2 integers: $3 - 6$
- Model the first integer.
- Take away the number of tiles equal to the second integer.

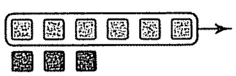
Model 3.

Adding zero pairs does not change the value. Zero pairs represent 0.

There are not enough tiles to take away 6.
To take away 6, we need 3 more tiles.
We add zero pairs. Add 3 tiles and 3 tiles.



Now take away the 6 tiles.



Since 3 tiles remain, we write: $3 - 6 = -3$

When tiles are not available, think of subtraction as the opposite of addition.
To subtract an integer, add its opposite integer.
For example,

$(-3) \underbrace{- (+2)}_{\text{Subtract } +2.} = -5$ $\qquad$ $(-3) \underbrace{+ (-2)}_{\text{Add } -2.} = -5$

Check

1. Subtract.

a) $(-6) - 2 =$ __−8__ $\qquad$ **b)** $2 - (-6) =$ __+8__

c) $(-8) - 9 =$ __−17__ $\qquad$ **d)** $8 - (-9) =$ __17__

Dividing Integers

When dividing 2 integers, look at the sign of each integer:

- When the integers have the same sign, their quotient is positive.
- When the integers have different signs, their quotient is negative.

The same rule applies to the multiplication of integers.

$6 \div (-3)$ These 2 integers have different signs, so their quotient is negative.

$6 \div (-3) = -2$

$(-10) \div (-2)$ These 2 integers have the same sign, so their quotient is positive.

$(-10) \div (-2) = 5$

Check

1. Calculate.

a) $(-4) \div 2$

$= \underline{-2}$

b) $(-6) \div (-3)$

$= \underline{2}$

c) $15 \div (-3)$

$= \underline{-5}$

TEACHER NOTE

For related review, see *Math Makes Sense 7*, Sections 2.2, 2.4, and 2.5; and *Math Makes Sense 8*, Sections 2.3, 2.4, and 2.5.

2.3 Order of Operations with Powers

FOCUS **Explain and apply the order of operations with exponents.**

We use this order of operations when evaluating an expression with powers:

- Do the operations in brackets first.
- Evaluate the powers.
- Multiply and divide, in order, from left to right.
- Add and subtract, in order, from left to right.

We can use the word BEDMAS to help us remember the order of operations:

B **B**rackets
E **E**xponents
D **D**ivision
M **M**ultiplication
A **A**ddition
S **S**ubtraction

Example 1 Adding and Subtracting with Powers

Evaluate.

a) $2^3 + 1$ **b)** $8 - 3^2$ **c)** $(3 - 1)^3$

Solution

a) $2^3 + 1$ — Evaluate the power first: 2^3
$= (2)(2)(2) + 1$ — Multiply: $(2)(2)(2)$
$= 8 + 1$ — Then add: $8 + 1$
$= 9$

b) $8 - 3^2$ — Evaluate the power first: 3^2
$= 8 - (3)(3)$ — Multiply: $(3)(3)$
$= 8 - 9$ — Then subtract: $8 - 9$
$= -1$

To subtract, add the opposite: $8 + (-9)$

c) $(3 - 1)^3$ — Subtract inside the brackets first: $3 - 1$
$= 2^3$ — Evaluate the power: 2^3
$= (2)(2)(2)$ — Multiply: $(2)(2)(2)$
$= 8$

Check

1. Evaluate.

a) $4^2 + 3 = \underline{(4)(4)} + 3$
$= \underline{16 + 3}$
$= \underline{19}$

b) $5^2 - 2^2 = \underline{(5)(5)} - (2)(2)$
$= \underline{25 - 4}$
$= \underline{21}$

c) $(2 + 1)^2 = \underline{3^2}$
$= \underline{(3)(3)}$
$= \underline{9}$

d) $(5 - 6)^2 = \underline{(-1)^2}$
$= \underline{(-1)(-1)}$
$= \underline{1}$

Example 2 Multiplying and Dividing with Powers

Evaluate.

a) $[2 \times (-2)^3]^2$

Curved brackets Square brackets

b) $(7^2 + 5^0) \div (-5)^1$

When we need curved brackets for integers, we use square brackets to show the order of operations.

Solution

a) $[2 \times (-2)^3]^2$ Evaluate what is inside the square brackets first: $2 \times (-2)^3$
$= [2 \times (-8)]^2$ Start with $(-2)^3 = -8$.
$= (-16)^2$
$= 256$

b) $(7^2 + 5^0) \div (-5)^1$ Evaluate what is inside the brackets first: $7^2 + 5^0$
$= (49 + 1) \div (-5)^1$ Add inside the brackets: $49 + 1$
$= 50 \div (-5)^1$ Evaluate the power: $(-5)^1$
$= 50 \div (-5)$
$= -10$

Check

1. Evaluate.

a) $5 \times 3^2 = 5 \times \underline{(3)}\ \underline{(3)}$
$= 5 \times \underline{9}$
$= \underline{45}$

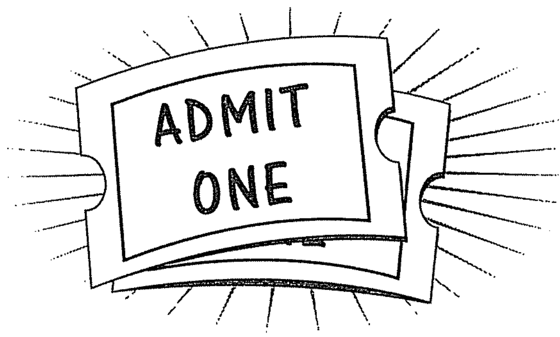
b) $8^2 \div 4 = \underline{(8)}\ \underline{(8)} \div 4$
$= \underline{64} \div 4$
$= \underline{16}$

c) $(3^2 + 6^0)^2 \div 2^1$
$= (\underline{9} + \underline{1})^2 \div 2^1$
$= \underline{10^2} \div 2^1$
$= \underline{100} \div \underline{2}$
$= \underline{50}$

d) $10^2 + (2 \times 2^2)^2 = 10^2 + (2 \times \underline{4})^2$
$= 10^2 + \underline{8^2}$
$= \underline{100} + \underline{64}$
$= \underline{164}$

Example 3 Solving Problems Using Powers

Corin answered the following skill-testing question to win free movie tickets:
$120 + 20^3 \div 10^3 + 12 \times 120$
His answer was 1568.
Did Corin win the movie tickets? Show your work.



Solution

$120 + 20^3 \div 10^3 + 12 \times 120$	Evaluate the powers first: 20^3 and 10^3
$= 120 + 8000 \div 1000 + 12 \times 120$	Divide and multiply.
$= 120 + 8 + 1440$	Add: $120 + 8 + 1440$
$= 1568$	

Corin won the movie tickets.

Check

1. Answer the following skill-testing question to enter a draw for a Caribbean cruise.

$(6 + 4) + 3^2 \times 10 - 10^2 \div 4$
$= \underline{10 + 9 \times 10 - 100 \div 4}$
$= \underline{10 + 90 - 25}$
$= \underline{75}$

Practice

1. Evaluate.

a) $2^2 + 1 = \underline{2 \times 2} + 1$
$= \underline{4} + 1$
$= \underline{5}$

b) $2^2 - 1 = \underline{2 \times 2} - 1$
$= \underline{4} - 1$
$= \underline{3}$

c) $(2 + 1)^2 = \underline{3^2}$
$= \underline{3 \times 3}$
$= \underline{9}$

d) $(2 - 1)^2 = \underline{1^2}$
$= \underline{1 \times 1}$
$= \underline{1}$

2. Evaluate.

a) $4 \times 2^2 = 4 \times \underline{2 \times 2}$
$= 4 \times \underline{4}$
$= \underline{16}$

b) $4^2 \times 2 = \underline{4 \times 4} \times 2$
$= \underline{16} \times 2$
$= \underline{32}$

c) $(4 \times 2)^2 = \underline{8^2}$
$= \underline{8 \times 8}$
$= \underline{64}$

d) $(-4)^2 \div 2 = \underline{(-4)(-4)} \div 2$
$= \underline{16} \div 2$
$= \underline{8}$

3. Evaluate.

a) $2^3 + (-1)^3 = \underline{(2)(2)(2)} + (-1)^3$
$= \underline{8} + (-1)^3$
$= \underline{8} + \underline{(-1)}\ \underline{(-1)}\ \underline{(-1)}$
$= \underline{8} + \underline{(-1)}$
$= \underline{7}$

b) $(2 - 1)^3 = \underline{1^3}$
$= \underline{(1)(1)(1)}$
$= \underline{1}$

c) $2^3 - (-1)^3 = \underline{(2)(2)(2)} - (-1)^3$
$= \underline{8} - (-1)^3$
$= \underline{8} - \underline{(-1)}\ \underline{(-1)}\ \underline{(-1)}$
$= \underline{8} - \underline{(-1)}$
$= \underline{9}$

d) $(2 + 1)^3 = \underline{3^3}$
$= \underline{(3)(3)(3)}$
$= \underline{27}$

4. Evaluate.

a) $3^2 \div (-1)^2 = \underline{(3)(3)} \div (-1)^2$
$= \underline{9} \div (-1)^2$
$= \underline{9} \div \underline{(-1)}\ \underline{(-1)}$
$= \underline{9} \div \underline{1}$
$= \underline{9}$

b) $(3 \div 1)^2 = \underline{3^2}$
$= \underline{3 \times 3}$
$= \underline{9}$

c) $3^2 \times (-2)^2 = \underline{(3)(3)} \times (-2)^2$
$= \underline{9} \times (-2)^2$
$= \underline{9} \times \underline{(-2)(-2)}$
$= \underline{9} \times \underline{4}$
$= \underline{36}$

d) $5^2 \div (-5)^1 = \underline{(5)(5)} \div (-5)^1$
$= \underline{25} \div (-5)^1$
$= \underline{25} \div \underline{(-5)}$
$= \underline{(-5)}$

5. Evaluate.

a) $(-2)^0 \times (-2) = \underline{1} \times (-2)$
$= \underline{(-2)}$

b) $2^3 \div (-2)^2 = \underline{(2)(2)(2)} \div (-2)^2$
$= \underline{8} \div (-2)^2$
$= \underline{8} \div \underline{(-2)(-2)}$
$= \underline{8} \div \underline{4}$
$= \underline{2}$

c) $(3 + 2)^0 + (3 \times 2)^0 = \underline{1} + \underline{1}$
$= \underline{2}$

d) $(3 \times 5^2)^0 = \underline{1}$

A power with exponent 0 is equal to 1.

e) $(2)(3) - (4)^2 = (2)(3) - \underline{(4)(4)}$
$= (2)(3) - \underline{16}$
$= \underline{6} - \underline{16}$
$= \underline{-10}$

f) $3(2 - 1)^2 = 3\underline{(1)^2}$
$= 3\underline{(1)}$
$= \underline{3}$

g) $(-2)^2 + (3)(4) = \underline{(-2)(-2)} + (3)(4)$
$= \underline{4} + (3)(4)$
$= \underline{4} + \underline{12}$
$= \underline{16}$

h) $(-2) + 3^0 \times (-2) = (-2) + \underline{1} \times (-2)$
$= (-2) + \underline{(-2)}$
$= \underline{-4}$

6. Amaya wants to replace the hardwood floor in her house.
Here is how she calculates the cost, in dollars:
$70 \times 6^2 + 60 \times 6^2$
How much will it cost Amaya to replace the hardwood floor?

$70 \times \underline{(6)(6)} + 60 \times \underline{(6)(6)}$

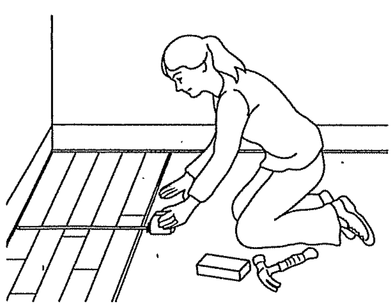
$= 70 \times \underline{36} + 60 \times \underline{36}$

$= \underline{2520} + \underline{2160}$

$= \underline{4680}$

It will cost Amaya $\$\underline{4680}$ to replace the hardwood floor.

Remember the order of operations: BEDMAS



TEACHER NOTE

Next Steps: Direct students to questions 6, 8, and 10 on page 66 of the Student Text.

For students experiencing success, introduce Example 3 on page 65 of the Student Text. Assign Practice questions 12, 13, and 19.

CHECKPOINT

Can you ...

- Use powers to show repeated multiplication?
- Use patterns to evaluate a power with exponent zero, such as 5^0?
- Use the correct order of operations with powers?

2.1 **1.** Give the base and exponent of each power.

a) 6^2 Base: **6** Exponent: **2**
There are **2** factors of **6**.

b) 4^5 Base: **4** Exponent: **5**
There are **5** factors of **4**.

c) $(-3)^8$ Base: **−3** Exponent: **8**
There are **8** factors of **(−3)**.

d) -3^8 Base: **3** Exponent: **8**
There are **8** factors of **3**.

2. Write as a power.

a) $7 \times 7 \times 7 \times 7 \times 7 \times 7 = 7^{6}$

b) $2 \times 2 \times 2 \times 2 = 2^{4}$

c) $5 = 5^1$

d) $(-5)(-5)(-5)(-5)(-5) = (-5)^5$

3. Write each power as repeated multiplication and in standard form.

a) $5^2 = 5 \times 5 = 25$

b) $2^3 = 2 \times 2 \times 2 = 8$

c) $3^4 = 3 \times 3 \times 3 \times 3 = 81$

2.2 **4. a)** Complete the table.

Power	Repeated Multiplication	Standard Form
7^3	$7 \times 7 \times 7$	343
7^2	7×7	**49**
7^1	**7**	**7**

b) What is the value of 7^0? **1**

5. Write each number in standard form and as a power of 10.

a) One hundred = 100 = 10^{2}

b) Ten thousand = **10 000** = 10^{4}

c) One million = **1 000 000** = 10^{6}

d) One = **1** = 10^{0}

6. Evaluate.

a) $6^0 =$ **1**

b) $(-8)^0 =$ **1**

c) $12^1 =$ **12**

d) $-8^0 =$ **−1**

7. Write each number in standard form.

a) 4×10^3
$= 4 \times$ **1000**
$=$ **4000**

b) $(1 \times 10^3) + (3 \times 10^2) + (2 \times 10^1) + (1 \times 10^0)$
$= (1 \times 1000) + (3 \times$ **100**$) + ($**2 × 10**$) + ($**1 × 1**$)$
$=$ **1000** + **300** + **20** + **1**
$=$ **1321**

c) $(4 \times 10^3) + (2 \times 10^2) + (3 \times 10^1) + (6 \times 10^0)$
$= (4 \times$ **1000**$) + ($**2 × 100**$) + ($**3 × 10**$) + ($**6 × 1**$)$
$=$ **4000** + **200** + **30** + **6**
$=$ **4236**

d) $(8 \times 10^2) + (1 \times 10^1) + (9 \times 10^0)$
$=$ **(8 × 100) + (1 × 10) + (9 × 1)**
$=$ **800 + 10 + 9**
$=$ **819**

2.3 **8.** Evaluate.

a) $3^2 + 5 = \underline{3 \times 3} + 5$
$= \underline{9} + 5$
$= \underline{14}$

b) $5^2 - 2^3 = \underline{5 \times 5} - 2^3$
$= \underline{25} - 2^3$
$= \underline{25} - \underline{(2)(2)(2)}$
$= \underline{25 - 8}$
$= \underline{17}$

c) $(2 + 3)^3 = (\underline{5})^3$
$= \underline{5 \times 5 \times 5}$
$= \underline{125}$

d) $2^3 + (-3)^3 = \underline{(2)(2)(2)} + (-3)^3$
$= \underline{8} + (-3)^3$
$= \underline{8} + \underline{(-3)(-3)(-3)}$
$= \underline{8 + (-27)}$
$= \underline{-19}$

9. Evaluate.

a) $5 \times 3^2 = 5 \times \underline{9}$
$= \underline{45}$

b) $8^2 \div 4 = \underline{64} \div 4$
$= \underline{16}$

c) $(10 + 2) \div 2^2 = \underline{12} \div 2^2$
$= \underline{12} \div \underline{4}$
$= \underline{3}$

d) $(7^2 + 1) \div (2^3 + 2)$
$= (\underline{49} + 1) \div (\underline{8} + 2)$
$= \underline{50} \div \underline{10}$
$= \underline{5}$

10. Evaluate. State which operation you do first.

a) $3^2 + 4^2$ **Exponents**
$= \underline{(3)(3)} + \underline{(4)(4)}$
$= \underline{9} + \underline{16}$
$= \underline{25}$

b) $[(-3) - 2]^3$ **Square brackets**
$= (\underline{-5})^3$
$= \underline{(-5)(-5)(-5)}$
$= \underline{-125}$

c) $(-2)^3 + (-3)^0$ **Exponents**
$= \underline{(-2)(-2)(-2)} + \underline{1}$
$= \underline{-8 + 1}$
$= \underline{-7}$

d) $[(6 - 3)^3 \times (2 + 2)^2]^0$ **Evaluate the 0 exponent**
$= \underline{1}$

2.4 Skill Builder

Simplifying Fractions

To simplify a fraction, divide the numerator and denominator by their common factors.

To simplify $\frac{5 \times 5 \times 5 \times 5}{5 \times 5}$:

This fraction shows repeated multiplication.

Divide the numerator and denominator by their common factors: 5×5.

$$\frac{\cancel{5}^{1} \times \cancel{5}^{1} \times 5 \times 5}{\cancel{5}^{1} \times \cancel{5}^{1}}$$

$$= \frac{5 \times 5}{1}$$

$$= 25$$

Check

1. Simplify each fraction.

What are the common factors?

TEACHER NOTE
For related review, see *Math Makes Sense 8*, Section 3.3.

a) $\frac{3 \times 3 \times 3}{3}$

$= \frac{\mathbf{3 \times 3}}{\mathbf{1}}$

$= \mathbf{9}$

b) $\frac{8 \times 8 \times 8 \times 8 \times 8}{8 \times 8 \times 8 \times 8 \times 8}$

$= \mathbf{1}$

c) $\frac{5 \times 5 \times 5 \times 5 \times 5}{5 \times 5 \times 5}$

$= \frac{\mathbf{5 \times 5}}{\mathbf{1}}$

$= \mathbf{25}$

d) $\frac{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2}$

$= \frac{\mathbf{2 \times 2 \times 2}}{\mathbf{1}}$

$= \mathbf{8}$

2.4 Exponent Laws I

FOCUS **Understand and apply the exponent laws for products and quotients of powers.**

Multiply $3^2 \times 3^4$.

$3^2 \times 3^4$ Write as repeated multiplication.

$= \underbrace{(3 \times 3)}_{\text{2 factors of 3}} \times \underbrace{(3 \times 3 \times 3 \times 3)}_{\text{4 factors of 3}}$

$= \underbrace{3 \times 3 \times 3 \times 3 \times 3 \times 3}_{\text{6 factors of 3}}$

$= 3^6$

Base Exponent

So, $3^2 \times 3^4 = 3^6$ Look at the pattern in the exponents.

2 + 4 = 6

We write: $3^{\mathbf{2}} \times 3^{\mathbf{4}} = 3^{\mathbf{(2 + 4)}}$

$= 3^{\mathbf{6}}$

This relationship is true when you multiply any 2 powers with the same base.

Exponent Law for a Product of Powers

To multiply powers with the same base, add the exponents.

Example 1 Simplifying Products with the Same Base

Write as a power.

a) $5^3 \times 5^4$ **b)** $(-6)^2 \times (-6)^3$ **c)** $(7^2)(7)$

Solution

a) The powers have the same base: 5

Use the exponent law for products: add the exponents.

$5^3 \times 5^4 = 5^{(3 + 4)}$

$= 5^7$

To check your work, you can write the powers as repeated multiplication.

b) The powers have the same base: -6

$(-6)^2 \times (-6)^3 = (-6)^{(2+3)}$ Add the exponents.

$= (-6)^5$

c) $(7^2)(7) = 7^2 \times 7^1$ Use the exponent law for products.

$= 7^{(2+1)}$ Add the exponents.

$= 7^3$

7 can be written as 7^1.

Check

1. Write as a power.

a) $2^5 \times 2^4 = 2^{(\underline{5} + \underline{4})}$
$= 2^{\underline{9}}$

b) $5^2 \times 5^5 = 5^{\underline{(2+5)}}$
$= 5^{\underline{7}}$

c) $(-3)^2 \times (-3)^3 = \underline{(-3)^{(2+3)}}$
$= \underline{(-3)^5}$

d) $10^5 \times 10 = \underline{10^{(5+1)}}$
$= \underline{10^6}$

Divide $3^4 \div 3^2$.

We can show division in fraction form.

$3^4 \div 3^2 = \frac{3^4}{3^2}$

$= \frac{3 \times 3 \times 3 \times 3}{3 \times 3}$ Simplify.

$= \frac{\cancel{3}^1 \times \cancel{3}^1 \times 3 \times 3}{\cancel{3}^1 \times \cancel{3}^1}$

$= \frac{3 \times 3}{1}$

$= 3 \times 3$

$= 3^2$

So, $3^4 \div 3^2 = 3^2$ Look at the pattern in the exponents.

$\mathbf{4} - \mathbf{2} = \mathbf{2}$

We write: $3^{\mathbf{4}} \div 3^{\mathbf{2}} = 3^{(\mathbf{4}-\mathbf{2})}$
$= 3^{\mathbf{2}}$

This relationship is true when you divide any 2 powers with the same base.

Exponent Law for a Quotient of Powers
To divide powers with the same base, subtract the exponents.

Example 2 Simplifying Quotients with the Same Base

Write as a power.

a) $4^5 \div 4^3$

b) $(-2)^7 \div (-2)^2$

Solution

Use the exponent law for quotients: subtract the exponents.

a) $4^5 \div 4^3 = 4^{(5-3)}$
$= 4^2$

The powers have the same base: 4

To check your work, you can write the powers as repeated multiplication.

b) $(-2)^7 \div (-2)^2 = (-2)^{(7-2)}$
$= (-2)^5$

The powers have the same base: -2

Check

1. Write as a power.

a) $(-5)^6 \div (-5)^3 = (-5)^{\underline{(6-3)}}$
$= \underline{(-5)^3}$

b) $\dfrac{(-3)^9}{(-3)^5} = (-3)^{\underline{(9-5)}}$
$= \underline{(-3)^4}$

$\frac{(-3)^9}{(-3)^5}$ is the same as $(-3)^9 \div (-3)^5$

c) $8^4 \div 8^3 = \underline{8^{(4-3)}}$
$= \underline{8^1}$

d) $9^8 \div 9^2 = \underline{9^{(8-2)}}$
$= \underline{9^6}$

Example 3 Evaluating Expressions Using Exponent Laws

Evaluate.

a) $2^2 \times 2^3 \div 2^4$

b) $(-2)^5 \div (-2)^3 \times (-2)$

Solution

a) $2^2 \times 2^3 \div 2^4$
$= 2^{(2+3)} \div 2^4$
$= 2^5 \div 2^4$
$= 2^{(5-4)}$
$= 2^1$
$= 2$

Add the exponents of the 2 powers that are multiplied.
Then, subtract the exponent of the power that is divided.

b) $(-2)^5 \div (-2)^3 \times (-2)$
$= (-2)^{(5-3)} \times (-2)$
$= (-2)^2 \times (-2)$
$= (-2)^{(2+1)}$
$= (-2)^{(3)}$
$= (-2)(-2)(-2)$
$= -8$

Subtract the exponents of the 2 powers that are divided.

Multiply: add the exponents.

TEACHER NOTE
If students are having difficulty, they should write the powers as repeated multiplication, and use brackets to visualize groupings of numbers.

Check

1. Evaluate.

a) $4 \times 4^3 \div 4^2 = 4^{(\underline{\mathbf{1}} + \underline{\mathbf{3}})} \div 4^2$
$= 4^{\underline{\mathbf{4}}} \div 4^2$
$= 4^{(\underline{\mathbf{4}} - \underline{\mathbf{2}})}$
$= 4^{\underline{\mathbf{2}}}$
$= \underline{\mathbf{16}}$

b) $(-3) \div (-3) \times (-3)$
$= (-3)^{\underline{\mathbf{(1-1)}}} \times (-3)$
$= (-3)^{\underline{\mathbf{0}}} \times (-3)$
$= (-3)^{\underline{\mathbf{(0+1)}}}$
$= (-3)^{\underline{\mathbf{1}}}$
$= \underline{\mathbf{-3}}$

$(-3) = (-3)^1$

Practice

1. Write each product as a single power.

a) $7^6 \times 7^2 = 7^{(\underline{6} + \underline{2})}$
$= 7^{\underline{8}}$

b) $(-4)^5 \times (-4)^3 = (-4)^{\underline{(5+3)}}$
$= (-4)^{\underline{8}}$

To multiply powers with the same base, add the exponents.

c) $(-2) \times (-2)^3 = \underline{(-2)^{(1+3)}}$
$= \underline{(-2)^4}$

d) $10^5 \times 10^5 = \underline{10^{(5+5)}}$
$= \underline{10^{10}}$

e) $7^0 \times 7^1 = \underline{7^{(0+1)}}$
$= \underline{7^1}$

f) $(-3)^4 \times (-3)^5 = \underline{(-3)^{(4+5)}}$
$= \underline{(-3)^9}$

2. Write each quotient as a power.

a) $(-3)^5 \div (-3)^2 = (-3)^{(\underline{5} - \underline{2})}$
$= (-3)^{\underline{3}}$

b) $5^6 \div 5^4 = 5^{\underline{(6-4)}}$
$= 5^{\underline{2}}$

To divide powers with the same base, subtract the exponents.

c) $\frac{4^7}{4^4} = 4^{\underline{(7-4)}}$
$= 4^{\underline{3}}$

d) $\frac{5^8}{5^6} = \underline{5^{(8-6)}}$
$= \underline{5^2}$

e) $6^4 \div 6^4 = \underline{6^{(4-4)}}$
$= \underline{6^0}$

f) $\frac{(-6)^8}{(-6)^7} = \underline{(-6)^{(8-7)}}$
$= \underline{(-6)^1}$

3. Write as a single power.

a) $2^3 \times 2^4 \times 2^5 = 2^{(\underline{3} + \underline{4})} \times 2^5$
$= 2^{\underline{7}} \times 2^5$
$= 2^{\underline{(7+5)}}$
$= 2^{\underline{12}}$

b) $\frac{3^2 \times 3^2}{3^2 \times 3^2} = \frac{3^{\underline{(2+2)}}}{3^{\underline{(2+2)}}}$
$= \frac{3^{\underline{4}}}{3^{\underline{4}}}$
$= \underline{3^{(4-4)}}$
$= \underline{3^0}$

Which exponent law should you use?

c) $10^3 \times 10^5 \div 10^2 = \underline{10^{(3+5)}} \div 10^2$
$= \underline{10^8} \div 10^2$
$= \underline{10^{(8-2)}}$
$= \underline{10^6}$

d) $(-1)^9 \div (-1)^5 \times (-1)^0$
$= \underline{(-1)^{(9-5)}} \times (-1)^0$
$= \underline{(-1)^4} \times (-1)^0$
$= \underline{(-1)^{(4+0)}}$
$= \underline{(-1)^4}$

4. Simplify, then evaluate.

a) $(-3)^1 \times (-3)^2 \times 2$
$= \mathbf{(-3)^{(1+2)}} \times 2$
$= \mathbf{(-3)^3} \times 2$
$= \mathbf{(-27)} \times 2$
$= \mathbf{-54}$

b) $9^9 \div 9^7 \times 9^0 = \mathbf{(9)^{(9-7)}} \times 9^0$
$= \mathbf{9^2} \times 9^0$
$= \mathbf{9^{(2+0)}}$
$= \mathbf{9^2}$
$= \mathbf{81}$

See if you can use the exponent laws to simplify.

c) $\frac{5^2}{5^0} = \mathbf{5^{(2-0)}}$
$= \mathbf{5^2}$
$= \mathbf{25}$

d) $\frac{5^5}{5^4} \times 5 = 5^{\mathbf{(5-4)}} \times 5$
$= 5^{\mathbf{1}} \times 5$
$= 5^{\mathbf{(1+1)}}$
$= 5^{\mathbf{2}}$
$= \mathbf{25}$

5. Identify any errors and correct them.

a) $4^3 \times 4^5 = 4^8$

$\mathbf{4^3 \times 4^5 = 4^{(3+5)}}$
$\mathbf{= 4^8}$

b) $2^5 \times 2^5 = 2^{25}$

$\mathbf{2^5 \times 2^5 = 2^{(5+5)}}$
$\mathbf{= 2^{10}}$**, not** $\mathbf{2^{25}}$

c) $(-3)^6 \div (-3)^2 = (-3)^3$

$\mathbf{(-3)^6 \div (-3)^2 = (-3)^{(6-2)}}$
$\mathbf{= (-3)^4}$**, not** $\mathbf{(-3)^3}$

d) $7^0 \times 7^2 = 7^0$

$\mathbf{7^0 \times 7^2 = 7^{(0+2)}}$
$\mathbf{= 7^2}$**, not** $\mathbf{7^0}$

e) $6^2 + 6^2 = 6^4$

$\mathbf{6^2 + 6^2 = (6)(6) + (6)(6)}$
$\mathbf{= 36 + 36}$
$\mathbf{= 72}$**, not** $\mathbf{6^4}$**,**
which is equal to 1296

f) $10^6 \div 10 = 10^6$

$\mathbf{10^6 \div 10 = 10^{(6-1)}}$
$\mathbf{= 10^5}$**, not** $\mathbf{10^6}$

g) $2^3 \times 5^2 = 10^5$

$\mathbf{2^3 \times 5^2 = (2)(2)(2) \times (5)(5)}$
$\mathbf{= 8 \times 25}$
$\mathbf{= 200}$**, not** $\mathbf{10^5}$**,**
which is equal to 100 000

TEACHER NOTE

Next Steps: Direct students to questions 6, 7, 8, and 9 on page 77 of the Student Text.

For students experiencing success, introduce Example 3 on Student Text page 76. Assign Practice questions 10, 11, 13, and 19.

2.5 Skill Builder

Grouping Equal Factors

In multiplication, you can group equal factors.

Order does not matter in multiplication.

For example:

$3 \times 7 \times 7 \times 3 \times 7 \times 7 \times 3$ Group equal factors.

$= \underbrace{3 \times 3 \times 3}_{} \times \underbrace{7 \times 7 \times 7 \times 7}_{}$ Write repeated multiplication as powers.

$= \quad 3^3 \quad \times \quad 7^4$

Check

1. Group equal factors and write as powers.

a) $2 \times 10 \times 2 \times 10 \times 2 = \mathbf{2 \times 2 \times 2 \times 10 \times 10}$
$= \mathbf{2^3 \times 10^2}$

b) $2 \times 5 \times 2 \times 5 \times 2 \times 5 \times 2 \times 5 = \mathbf{2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5}$
$= \mathbf{2^4 \times 5^4}$

Multiplying Fractions

To multiply fractions, first multiply the numerators, and then multiply the denominators.

$\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{2 \times 2 \times 2 \times 2}{3 \times 3 \times 3 \times 3}$ Write repeated multiplication as powers.

$= \frac{2^4}{3^4}$

There are 4 factors of 2, and 4 factors of 3.

Check

1. Multiply the fractions. Write as powers.

a) $\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \mathbf{\frac{3 \times 3 \times 3}{4 \times 4 \times 4}}$
$= \mathbf{\frac{3^3}{4^3}}$

b) $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$
$= \mathbf{\frac{1 \times 1 \times 1 \times 1 \times 1 \times 1}{2 \times 2 \times 2 \times 2 \times 2 \times 2}}$
$= \mathbf{\frac{1^6}{2^6}}$

TEACHER NOTE
For related review, see *Math Makes Sense 8*, Section 3.3.

2.5 Exponent Laws II

FOCUS **Understand and apply exponent laws for powers of: products; quotients; and powers.**

Multiply $3^2 \times 3^2 \times 3^2$.

$3^2 \times 3^2 \times 3^2 = 3^{2+2+2}$

$= 3^6$

Use the exponent law for the product of powers. Add the exponents.

We can write repeated multiplication as powers.

So, $\underbrace{3^2 \times 3^2 \times 3^2}_{\text{3 factors of } (3^2)}$

The base is 3^2. This is also a power.

The exponent is 3.

$= (3^2)^3$

This is a **power of a power.**

$= 3^6$

Look at the pattern in the exponents.

$\mathbf{2 \times 3 = 6}$

We write: $(3^2)^3 = 3^{2 \times 3}$

$= 3^6$

Exponent Law for a Power of a Power

To raise a power to a power, multiply the exponents.

For example: $(2^3)^5 = 2^{3 \times 5}$

Example 1 Simplifying a Power of a Power

Write as a power.

a) $(3^2)^4$ **b)** $[(-5)^3]^2$ **c)** $-(2^3)^4$

Solution

Use the exponent law for a power of a power: multiply the exponents.

a) $(3^2)^4 = 3^{2 \times 4}$

$= 3^8$

b) $[(-5)^3]^2 = (-5)^{3 \times 2}$

$= (-5)^6$

The base is -5.

c) $-(2^3)^4 = -(2^{3 \times 4})$

$= -2^{12}$

The base is 2.

Check

1. Write as a power.

a) $(9^3)^4 = 9^{\underline{3} \times \underline{4}}$
$= 9^{\underline{12}}$

b) $[(-2)^5]^3 = (-2)^{\underline{5 \times 3}}$
$= (-2)^{\underline{15}}$

c) $-(5^4)^2 = -(5^{\underline{4 \times 2}})$
$= -5^{\underline{8}}$

Multiply $(3 \times 4)^2$.

The base of the power is a product: $\underbrace{3 \times 4}_{\textbf{base}}$

Write as repeated multiplication.

$(3 \times 4)^2 = (3 \times 4) \times (3 \times 4)$
$= 3 \times 4 \times 3 \times 4$ Remove the brackets.
$= (\underbrace{3 \times 3}_{\text{2 factors of 3}}) \times (\underbrace{4 \times 4}_{\text{2 factors of 4}})$ Group equal factors.
$= 3^2 \times 4^2$ Write as powers.

So, $(3 \times 4)^2 = 3^2 \times 4^2$

power product power

Exponent Law for a Power of a Product

The power of a product is the product of powers.
For example: $(2 \times 3)^4 = 2^4 \times 3^4$

Example 2 Evaluating Powers of Products

Evaluate.

a) $(2 \times 5)^2$

b) $[(-3) \times 4]^2$

Solution

Use the exponent law for a power of a product.

a) $(2 \times 5)^2 = 2^2 \times 5^2$
$= (2)(2) \times (5)(5)$
$= 4 \times 25$
$= 100$

b) $[(-3) \times 4]^2 = (-3)^2 \times 4^2$
$= (-3)(-3) \times (4)(4)$
$= 9 \times 16$
$= 144$

Or, use the order of operations and evaluate what is inside the brackets first.

a) $(2 \times 5)^2 = 10^2$
$= 100$

b) $[(-3) \times 4]^2 = (-12)^2$
$= 144$

Check

1. Write as a product of powers.

a) $(5 \times 7)^4 = \underline{5^4} \times \underline{7^4}$

b) $(8 \times 2)^2 = \underline{8^2} \times \underline{2^2}$

2. Evaluate.

a) $[(-1) \times 6]^2 = \underline{(-6)^2}$
$= \underline{36}$

b) $[(-1) \times (-4)]^3 = \underline{4^3}$
$= \underline{64}$

Evaluate $\left(\underbrace{\frac{3}{4}}_{\textbf{base}}\right)^2$.

The base of the power is a quotient: $\frac{3}{4}$

Write as repeated multiplication.

$$\left(\frac{3}{4}\right)^2 = \left(\frac{3}{4}\right) \times \left(\frac{3}{4}\right)$$

$$= \frac{3}{4} \times \frac{3}{4}$$

Multiply the fractions.

$$= \frac{3 \times 3}{4 \times 4}$$

Write repeated multiplication as powers.

$$= \frac{3^2}{4^2}$$

So, $\left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2}$

power

quotient

power

Exponent Law for a Power of a Quotient

The power of a quotient is the quotient of powers.

For example: $\left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4}$

Example 3 Evaluating Powers of Quotients

Evaluate.

a) $[30 \div (-5)]^2$

b) $\left(\frac{20}{4}\right)^2$

Solution

Use the exponent law for a power of a quotient.

a) $[30 \div (-5)]^2 = \left(\frac{30}{-5}\right)^2$

$= \frac{30^2}{(-5)^2}$

$= \frac{900}{25}$

$= 36$

b) $\left(\frac{20}{4}\right)^2 = \frac{20^2}{4^2}$

$= \frac{400}{16}$

$= 25$

Or, use the order of operations and evaluate what is inside the brackets first.

a) $[30 \div (-5)]^2 = (-6)^2$

$= 36$

b) $\left(\frac{20}{4}\right)^2 = 5^2$

$= 25$

Check

1. Write as a quotient of powers.

a) $\left(\frac{3}{4}\right)^5 = \frac{3^5}{4^5}$

b) $[1 \div (-10)]^3 = \frac{1^3}{(-10)^3}$

2. Evaluate.

a) $[(-16) \div (-4)]^2$

$= 4^2$ $= 16$

b) $\left(\frac{36}{6}\right)^3 = 6^3$

$= 216$

You can evaluate what is inside the brackets first.

Practice

1. Write as a product of powers.

a) $(5 \times 2)^4 = 5^4 \times 2^4$

b) $(12 \times 13)^2 = 12^2 \times 13^2$

c) $[3 \times (-2)]^3 = 3^3 \times (-2)^3$

d) $[(-4) \times (-5)]^5 = (-4)^5 \times (-5)^5$

2. Write as a quotient of powers.

a) $(5 \div 8)^0 = \frac{5^0}{8^0}$

b) $[(-6) \div 5]^7 = \frac{(-6)^7}{5^7}$

c) $\left(\frac{3}{5}\right)^2 = \frac{3^2}{5^2}$

d) $\left(\frac{-1}{-2}\right)^3 = \frac{(-1)^3}{(-2)^3}$

3. Write as a power.

a) $(5^2)^3 = 5^{2 \times 3}$
$= 5^6$

b) $[(-2)^3]^5 = (-2)^{3 \times 5}$
$= (-2)^{15}$

c) $(4^4)^1 = 4^{4 \times 1}$
$= 4^4$

d) $(8^0)^3 = 8^{0 \times 3}$
$= 8^0$

4. Evaluate.

a) $[(6 \times (-2)]^2 = (-12)^2$
$= 144$

b) $-(3 \times 4)^2 = -(12)^2$
$= -144$

c) $\left(\frac{-8}{-2}\right)^2 = 4^2$
$= 16$

d) $(10 \times 3)^1 = 30^1$
$= 30$

e) $[(-2)^1]^2 = (-2)^{1 \times 2}$
$= (-2)^2$
$= 4$

f) $[(-2)^1]^3 = (-2)^{1 \times 3}$
$= (-2)^3$
$= -8$

5. Find any errors and correct them.

a) $(3^2)^3 = 3^5$

$(3^2)^3 = 3^{2 \times 3}$
$= 3^6$, not 3^5

b) $(3 + 2)^2 = 3^2 + 2^2$

$(3 + 2)^2 = 5^2$
$= 25$, not $3^2 + 2^2$ which is equal to 13

c) $(5^3)^3 = 5^9$

$(5^3)^3 = 5^{3 \times 3}$
$= 5^9$

d) $\left(\frac{2}{3}\right)^8 = \frac{2^8}{3^8}$

$\left(\frac{2}{3}\right)^8 = \frac{2^8}{3^8}$

e) $(3 \times 2)^2 = 36$

$(3 \times 2)^2 = 6^2$
$= 36$

f) $\left(\frac{2}{3}\right)^2 = \frac{4}{6}$

$\left(\frac{2}{3}\right)^2 = \frac{2^2}{3^2}$
$= \frac{4}{9}$, not $\frac{4}{6}$

g) $[(-3)^3]^0 = (-3)^3$

$[(-3)^3]^0 = (-3)^{3 \times 0}$
$= (-3)^0$, not $(-3)^3$

h) $[(-2) \times (-3)]^4 = -6^4$

$[(-2) \times (-3)]^4 = 6^4$, not -6^4

TEACHER NOTE

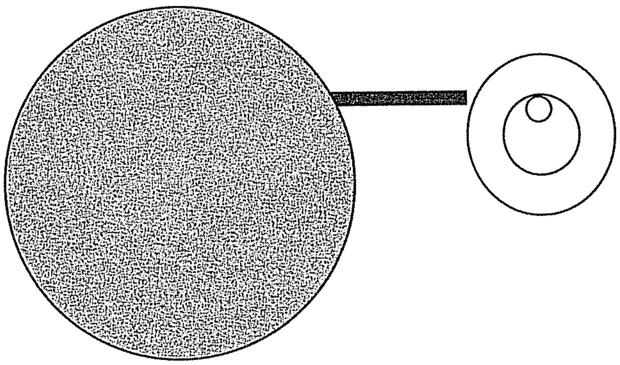
Next Steps: Direct students to questions 7, 8, and 10 on page 84 of the Student Text.

For students experiencing success, introduce Example 3 on page 83 of the Student Text. Assign Practice questions 14, 16, 17, and 19.

Unit 2 Puzzle

Bird's Eye View

This is a view through the eyes of a bird. What does the bird see?



To find out, simplify or evaluate each expression on the left, then find the answer on the right.
Write the corresponding letter beside the question number.
The numbers at the bottom of the page are question numbers.
Write the corresponding letter over each number.

1. $5 \times 5 \times 5 \times 5$	$= 5^4$ (R)	A	100 000
2. 2^3	$= 8$ (I)	P	5^6
3. $\frac{3^6}{3^2}$	$= 3^4$ (F)	S	0
4. $4 \times 4 \times 4 \times 4 \times 4$	$= 4^5$ (N)	E	1
5. $(-2)^3$	$= -8$ (Y)	F	3^4
6. $(-2) + 4 \div 2$	$= 0$ (S)	G	6
7. $(5^2)^3$	$= 5^6$ (P)	I	8
8. $3^2 - 2^3$	$= 1$ (E)	O	4^6
9. $10^2 \times 10^3$	$= 100\ 000$ (A)	N	4^5
10. $5 + 3^0$	$= 6$ (G)	R	5^4
11. $4^7 \div 4$	$= 4^6$ (O)	Y	-8

A	P	E	R	S	O	N	F	R	Y	I	N	G	A	N	E	G	G
9	7	8	1	6	11	4	3	1	5	2	4	10	9	4	8	10	10

Unit 2 Study Guide

Skill	Description	Example
Evaluate a power with an integer base.	Write the power as repeated multiplication, then evaluate.	$(-2)^3 = (-2) \times (-2) \times (-2)$ $= -8$
Evaluate a power with an exponent 0.	A power with an integer base and an exponent 0 is equal to 1.	$8^0 = 1$
Use the order of operations to evaluate expressions containing exponents.	Evaluate what is inside the brackets. Evaluate powers. Multiply and divide, in order, from left to right. Add and subtract, in order, from left to right.	$(3^2 + 2) \times (-5)$ $= (9 + 2) \times (-5)$ $= (11) \times (-5)$ $= -55$
Apply the exponent law for a product of powers.	To multiply powers with the same base, add the exponents.	$4^3 \times 4^6 = 4^{3+6}$ $= 4^9$
Apply the exponent law for a quotient of powers.	To divide powers with the same base, subtract the exponents.	$2^7 \div 2^4 = \frac{2^7}{2^4}$ $= 2^{7-4}$ $= 2^3$
Apply the exponent law for a power of a power.	To raise a power to a power, multiply the exponents.	$(5^3)^2 = 5^{3 \times 2}$ $= 5^6$
Apply the exponent law for a power of a product.	Write the power of a product as a product of powers.	$(6 \times 3)^5 = 6^5 \times 3^5$
Apply the exponent law for a power of a quotient.	Write the power of a quotient as a quotient of powers.	$\left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2}$

Unit 2 Review

2.1 **1.** Give the base and exponent of each power.

a) 6^2 Base $\underline{6}$ Exponent $\underline{2}$

b) $(-3)^8$ Base $\underline{-3}$ Exponent $\underline{8}$

2. Write as a power.

a) $4 \times 4 \times 4 = \underline{4^3}$

b) $(-3)(-3)(-3)(-3)(-3) = \underline{(-3)^5}$

3. Write each power as repeated multiplication and in standard form.

a) $(-2)^5 = \underline{(-2)(-2)(-2)(-2)(-2)}$
$= \underline{-32}$

b) $10^4 = \underline{10 \times 10 \times 10 \times 10}$
$= \underline{10\ 000}$

c) Six squared $= \underline{6^2}$
$= \underline{6 \times 6}$
$= \underline{36}$

d) Five cubed $= \underline{5^3}$
$= \underline{5 \times 5 \times 5}$
$= \underline{125}$

2.2 **4.** Evaluate.

a) $10^0 = \underline{1}$

b) $(-4)^0 = \underline{1}$

c) $8^1 = \underline{8}$

d) $-4^0 = \underline{-1}$

5. Write each number in standard form.

a) 9×10^3
$= 9 \times \underline{10} \times \underline{10} \times \underline{10}$
$= 9 \times \underline{1000}$
$= \underline{9000}$

b) $(1 \times 10^2) + (3 \times 10^1) + (5 \times 10^0)$
$= (1 \times \underline{100}) + (3 \times \underline{10}) + (5 \times \underline{1})$
$= \underline{\mathbf{100 + 30 + 5}}$
$= \underline{\mathbf{135}}$

c) $(2 \times 10^3) + (4 \times 10^2) + (1 \times 10^1) + (9 \times 10^0)$
$= (2 \times \underline{1000}) + (4 \times \underline{100}) + (1 \times \underline{10}) + (9 \times \underline{1})$
$= \underline{\mathbf{2000 + 400 + 10 + 9}}$
$= \underline{\mathbf{2419}}$

d) $(5 \times 10^4) + (3 \times 10^2) + (7 \times 10^1) + (2 \times 10^0)$
$= \underline{\mathbf{(5 \times 10\,000) + (3 \times 100) + (7 \times 10) + (2 \times 1)}}$
$= \underline{\mathbf{50\,000 + 300 + 70 + 2}}$
$= \underline{\mathbf{50\,372}}$

2.3 **6.** Evaluate.

a) $3^2 + 3$
$= \underline{\mathbf{3 \times 3}} + 3$
$= \underline{\mathbf{9}} + 3$
$= \underline{\mathbf{12}}$

b) $[(-2) + 4)]^3$
$= \underline{\mathbf{2^3}}$
$= \underline{\mathbf{2 \times 2 \times 2}}$
$= \underline{\mathbf{8}}$

c) $(20 + 5) \div 5^2 = \underline{\mathbf{25}} \div 5^2$
$= \underline{\mathbf{25}} \div \underline{\mathbf{25}}$
$= \underline{\mathbf{1}}$

d) $(8^2 - 4) \div (6^2 - 6)$
$= (\underline{\mathbf{64}} - 4) \div (\underline{\mathbf{36}} - 6)$
$= \underline{\mathbf{60}} \div \underline{\mathbf{30}}$
$= \underline{\mathbf{2}}$

7. Evaluate.

a) $5 \times 3^2 = 5 \times \underline{\mathbf{9}}$
$= \underline{\mathbf{45}}$

b) $10 \times (3^2 + 5^0) = 10 \times \underline{\mathbf{(9 + 1)}}$
$= 10 \times \underline{\mathbf{10}}$
$= \underline{\mathbf{100}}$

c) $(-2)^3 + (-3)(4) = \underline{\mathbf{(-8)}} + \underline{\mathbf{(-12)}}$
$= \underline{\mathbf{-20}}$

d) $(-3) + 4^0 \times (-3) = (-3) + \underline{\mathbf{1}} \times (-3)$
$= (-3) + \underline{\mathbf{(-3)}}$
$= \underline{\mathbf{-6}}$

2.4 **8.** Write as a power.

a) $6^3 \times 6^7 = 6^{(\underline{3} + \underline{7})}$
$= 6^{\underline{10}}$

b) $(-4)^2 \times (-4)^3 = (-4)^{\underline{(2+3)}}$
$= (-4)^{\underline{5}}$

c) $(-2)^5 \times (-2)^4 = (-2)^{\underline{(5+4)}}$
$= (-2)^{\underline{9}}$

d) $10^7 \times 10 = \underline{\mathbf{10^{(7+1)}}}$
$= \underline{\mathbf{10^8}}$

9. Write as a power.

a) $5^7 \div 5^3 = 5^{(\underline{7} - \underline{3})}$
$= 5^{\underline{4}}$

b) $\dfrac{10^5}{10^3} = \underline{\mathbf{10^{(5-3)}}}$
$= \underline{\mathbf{10^2}}$

c) $(-6)^8 \div (-6)^2 = \underline{\mathbf{(-6)^{(8-2)}}}$
$= \underline{\mathbf{(-6)^6}}$

d) $\dfrac{5^{10}}{5^6} = \underline{\mathbf{5^{(10-6)}}}$
$= \underline{\mathbf{5^4}}$

e) $8^3 \div 8 = \underline{\mathbf{8^{(3-1)}}}$
$= \underline{\mathbf{8^2}}$

f) $\dfrac{(-3)^4}{(-3)^0} = \underline{\mathbf{(-3)^{(4-0)}}}$
$= \underline{\mathbf{(-3)^4}}$

2.5 **10.** Write as a power.

a) $(5^3)^4 = 5^{\underline{3} \times \underline{4}}$
$= 5^{\underline{12}}$

b) $[(-3)^2]^6 = (-3)^{\underline{2} \times \underline{6}}$
$= (-3)^{\underline{12}}$

c) $(8^2)^4 = \underline{\mathbf{8^{2 \times 4}}}$
$= \underline{\mathbf{8^8}}$

d) $[(-5)^5]^4 = \underline{\mathbf{(-5)^{5 \times 4}}}$
$= \underline{\mathbf{(-5)^{20}}}$

11. Write as a product or quotient of powers.

a) $(3 \times 5)^2 = 3^{\underline{2}} \times 5^{\underline{2}}$

b) $(2 \times 10)^5 = \underline{\mathbf{2^5 \times 10^5}}$

c) $[(-4) \times (-5)]^3 = \underline{\mathbf{(-4)^3 \times (-5)^3}}$

d) $\left(\dfrac{4}{3}\right)^5 = \underline{\dfrac{\mathbf{4^5}}{\mathbf{3^5}}}$

e) $(12 \div 10)^4 = 12^{\underline{4}} \div 10^{\underline{4}}$

f) $[(-7) \div (-9)]^6 = \underline{\mathbf{(-7)^6 \div (-9)^6}}$