

UNIT
2

Powers and Exponent Laws

What You'll Learn

- Use powers to show repeated multiplication.
- Evaluate powers with exponent 0.
- Write numbers using powers of 10.
- Use the order of operations with exponents.
- Use the exponent laws to simplify and evaluate expressions.

Why It's Important

Powers are used by

- lab technicians, when they interpret a patient's test results
- reporters, when they write large numbers in a news story

Key Words

integer	exponent
opposite	squared
positive	cubed
negative	standard form
factor	product
power	quotient
base	

2.1 Skill Builder

Multiplying Integers

When multiplying 2 integers, look at the sign of each integer:

- When the integers have the same sign, their product is positive.
- When the integers have different signs, their product is negative.

\times	$(-)$	$(+)$
$(-)$	$(+)$	$(-)$
$(+)$	$(-)$	$(+)$

$6 \times (-3)$ These 2 integers have different signs, so their product is negative.

$$6 \times (-3) = -18$$

$(-10) \times (-2)$ These 2 integers have the same sign, so their product is positive.

$$(-10) \times (-2) = 20$$

When an integer is positive, we do not have to write the + sign in front.

Check

1. Will the product be positive or negative?

a) 7×4 _____

b) $3 \times (-6)$ _____

c) $(-9) \times 10$ _____

d) $(-5) \times (-9)$ _____

2. Multiply.

a) $7 \times 4 =$ _____

b) $3 \times (-6) =$ _____

c) $(-9) \times 10 =$ _____

d) $(-5) \times (-9) =$ _____

e) $(-3) \times (-5) =$ _____

f) $2 \times (-5) =$ _____

g) $(-8) \times 2 =$ _____

h) $(-4) \times 3 =$ _____

Multiplying More than 2 Integers

We can multiply more than 2 integers.

Multiply pairs of integers, from left to right.

$$\begin{aligned}(-1) \times (-2) \times (-3) \\ &= 2 \times (-3) \\ &= -6\end{aligned}$$

$$\begin{aligned}(-1) \times (-2) \times (-3) \times (-4) \\ &= 2 \times (-3) \times (-4) \\ &= (-6) \times (-4) \\ &= 24\end{aligned}$$

The product of 3 negative factors is negative.

The product of 4 negative factors is positive.

Multiplying Integers

When the number of negative factors is *even*, the product is positive.

When the number of negative factors is *odd*, the product is negative.

We can show products of integers in different ways:

$(-2) \times (-2) \times 3 \times (-2)$ is the same as $(-2)(-2)(3)(-2)$.

$$\begin{aligned}\text{So, } (-2) \times (-2) \times 3 \times (-2) &= (-2)(-2)(3)(-2) \\ &= -24\end{aligned}$$

Check

1. Multiply.

a) $(-3) \times (-2) \times (-1) \times 1$ _____

b) $(-2)(-1)(-2)(-2)(2)$ _____

c) $(-2)(-2)(-1)(-2)(-2)$ _____

d) $3 \times 3 \times 2$ _____

Is the answer positive or negative? How can you tell?

2.1 What Is a Power?

FOCUS Show repeated multiplication as a power.

We can use powers to show repeated multiplication.

$$2 \times 2 \times 2 \times 2 \times 2 = 2^5$$

↑ Repeated multiplication
5 factors of 2
↑ Power

2 is the **base**.
 5 is the **exponent**.
 2^5 is a **power**.

We read 2^5 as "2 to the 5th."

Here are some other powers of 2.

Repeated Multiplication	Power	Read as...
$\underbrace{2}$ 1 factor of 2	2^1	2 to the 1st
$\underbrace{2 \times 2}$ 2 factors of 2	2^2	2 to the 2nd, or 2 squared
$\underbrace{2 \times 2 \times 2}$ 3 factors of 2	2^3	2 to the 3rd, or 2 cubed
$\underbrace{2 \times 2 \times 2 \times 2}$ 4 factors of 2	2^4	2 to the 4th

In each case, the exponent in the power is equal to the number of factors in the repeated multiplication.

Example 1 Writing Powers

Write as a power.

a) $4 \times 4 \times 4 \times 4 \times 4 \times 4$

b) 3

Solution

a) The base is 4.

$$\underbrace{4 \times 4 \times 4 \times 4 \times 4 \times 4}_{6 \text{ factors of } 4} = 4^6$$

So, $4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^6$

b) The base is 3.

$$\underbrace{3}_{1 \text{ factor of } 3}$$

So, $3 = 3^1$

Check

1. Write as a power.

a) $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2$ —

b) $5 \times 5 \times 5 \times 5 = 5$ —

c) $(-10)(-10)(-10) =$ _____

d) $4 \times 4 =$ _____

e) $(-7)(-7)(-7)(-7)(-7)(-7)(-7)(-7) =$ _____

2. Complete the table.

	Repeated Multiplication	Power	Read as...
a)	$8 \times 8 \times 8 \times 8$	_____	8 to the 4th
b)	7×7	_____	_____
c)	$3 \times 3 \times 3 \times 3 \times 3 \times 3$	_____	3 to the 6th
d)	$2 \times 2 \times 2$	_____	_____

Power	Repeated Multiplication	Standard Form
2^5	$2 \times 2 \times 2 \times 2 \times 2$	32

Example 2 Evaluating Powers

Write as repeated multiplication and in standard form.

a) 2^4

b) 5^3

Solution

a) $2^4 = 2 \times 2 \times 2 \times 2$
 $= 16$

As repeated multiplication
Standard form

b) $5^3 = 5 \times 5 \times 5$
 $= 125$

As repeated multiplication
Standard form

Check

1. Complete the table.

Power	Repeated Multiplication	Standard Form
2^3	$2 \times 2 \times 2$	_____
6^2	_____	36
3^4	_____	_____
10^4	_____	_____
8 squared	_____	_____
7 cubed	_____	_____

To evaluate a power that contains negative integers, identify the base of the power. Then, apply the rules for multiplying integers.

Example 3 Evaluating Expressions Involving Negative Signs

Identify the base, then evaluate each power.

a) $(-5)^4$

b) -5^4

Solution

a) $(-5)^4$

$$\begin{aligned}(-5)^4 &= (-5) \times (-5) \times (-5) \times (-5) \\ &= 625\end{aligned}$$

The brackets tell us that the base of this power is (-5) .

There is an even number of negative integers, so the product is positive.

b) -5^4

$$\begin{aligned}-5^4 &= -(5 \times 5 \times 5 \times 5) \\ &= -625\end{aligned}$$

There are no brackets. So, the base of this power is 5. The negative sign applies to the whole expression.

Check

1. Identify the base of each power, then evaluate.

a) $(-1)^3$

The base is _____.

$$\begin{aligned}(-1)^3 &= \underline{\hspace{2cm}} \\ &= \underline{\hspace{2cm}}\end{aligned}$$

b) -10^3

The base is _____.

$$\begin{aligned}-10^3 &= \underline{\hspace{2cm}} \\ &= \underline{\hspace{2cm}}\end{aligned}$$

c) $(-7)^2$

The base is _____.

$$\begin{aligned}(-7)^2 &= \underline{\hspace{2cm}} \\ &= \underline{\hspace{2cm}}\end{aligned}$$

d) $-(-5)^4$

The base is _____.

$$\begin{aligned}-(-5)^4 &= \underline{\hspace{2cm}} \\ &= \underline{\hspace{2cm}}\end{aligned}$$

The first negative sign applies to the whole expression.

Practice

1. Write as a power.

a) $\underbrace{8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8}_{7 \text{ factors of } 8}$

The base is 8. There are _____ equal factors, so the exponent is _____.

$$8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 = 8 \underline{\hspace{1cm}}$$

b) $\underbrace{10 \times 10 \times 10 \times 10 \times 10}_{5 \text{ factors of } 10}$

The base is _____. There are _____ equal factors, so the exponent is _____.

$$\text{So, } 10 \times 10 \times 10 \times 10 \times 10 = \underline{\hspace{2cm}}$$

c) $\underbrace{(-2)(-2)(-2)}_{3 \text{ factors of } \underline{\hspace{1cm}}}$

The base is _____. There are _____ equal factors, so the exponent is _____.

$$\text{So, } (-2)(-2)(-2) = \underline{\hspace{2cm}}$$

d) $\underbrace{(-13)(-13)(-13)(-13)(-13)(-13)}_{\underline{\hspace{1cm}} \text{ factors of } \underline{\hspace{1cm}}}$

The base is _____. There are _____ equal factors, so the exponent is _____.

$$\text{So, } (-13)(-13)(-13)(-13)(-13)(-13) = \underline{\hspace{2cm}}$$

2. Write each expression as a power.

a) $9 \times 9 \times 9 \times 9 = \underline{\hspace{1cm}}^4$

b) $(5)(5)(5)(5)(5)(5) = 5 \underline{\hspace{1cm}}$

c) $11 \times 11 = \underline{\hspace{1cm}}$

d) $(-12)(-12)(-12)(-12)(-12) = \underline{\hspace{2cm}}$

3. Write each power as repeated multiplication.

a) $3^2 =$ _____

b) $3^4 =$ _____

c) $2^7 =$ _____

d) $10^8 =$ _____

Identify the base first.

4. State whether the answer will be positive or negative.

a) $(-3)^2$ _____

b) 6^3 _____

c) $(-10)^3$ _____

d) -4^3 _____

5. Write each power as repeated multiplication and in standard form.

a) $(-3)^2 =$ _____
= _____

b) $6^3 =$ _____
= _____

c) $(-10)^3 =$ _____
= _____

d) $-4^3 =$ _____
= _____

Predict. Will the answer be positive or negative?

6. Write each product as a power and in standard form.

a) $(-3)(-3)(-3) =$ _____
= _____

b) $(-8)(-8) =$ _____
= _____

c) $-(8 \times 8 \times 8) =$ _____
= _____

d) $-(-1)(-1)(-1)(-1)(-1)(-1)(-1) =$ _____
= _____

7. Identify any errors and correct them.

a) $4^3 = 12$ _____

b) $(-2)^9$ is negative. _____

c) $(-9)^2$ is negative. _____

d) $3^2 = 2^3$ _____

e) $(-10)^2 = 100$ _____

2.2 Skill Builder

Patterns and Relationships in Tables

Look at the patterns in this table.

Input		Output
1	$\times 2$	2
2	$\times 2$	4
3	$\times 2$	6
4	$\times 2$	8
5	$\times 2$	10

Diagram annotations: On the left, four upward-pointing curved arrows between rows are labeled '+1', indicating the input increases by 1. On the right, four downward-pointing curved arrows between rows are labeled '+2', indicating the output increases by 2. Horizontal arrows point from the input column to the output column, with a ' $\times 2$ ' label above each arrow.

The input starts at 1 and increases by 1 each time.

The output starts at 2 and increases by 2 each time.

The input and output are also related.

Double the input to get the output.

Check

1. a) Describe the patterns in the table.
b) What is the input in the last row?
What is the output in the last row?

Input	Output
1	5
2	10
3	15
4	20
_____	_____

Diagram annotations: On the left, four downward-pointing curved arrows between rows are labeled '+1', indicating the input increases by 1. On the right, four downward-pointing curved arrows between rows are labeled '+5', indicating the output increases by 5.

- a) The input starts at _____, and increases by _____ each time.
The output starts at _____, and increases by _____ each time.
You can also multiply the input by _____ to get the output.
- b) The input in the last row is $4 + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$.
The output in the last row is $20 + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$.

2. a) Describe the patterns in the table.

b) Extend the table 3 more rows.

Input	Output
10	100
9	90
8	80
7	70
6	60

a) The input starts at 10, and decreases by _____ each time.
The output starts at 100, and decreases by _____ each time.
You can also multiply the input by _____ to get the output.

b) To extend the table 3 more rows, continue to decrease the input by _____ each time.
Decrease the output by _____ each time.

Input	Output
5	_____
_____	_____
_____	_____

Writing Numbers in Expanded Form

8000 is 8 thousands, or 8×1000

600 is 6 hundreds, or 6×100

50 is 5 tens, or 5×10

Read it aloud.

Check

1. Write each number in expanded form.

a) 7000 _____

b) 900 _____

c) 400 _____

d) 30 _____

2.2 Powers of Ten and the Zero Exponent

FOCUS Explore patterns and powers of 10 to develop a meaning for the exponent 0.

This table shows decreasing powers of 3.

Power	Repeated Multiplication	Standard Form
3^5	$3 \times 3 \times 3 \times 3 \times 3$	243
3^4	$3 \times 3 \times 3 \times 3$	81
3^3	$3 \times 3 \times 3$	27
3^2	3×3	9
3^1	3	3

Look for patterns in the columns.

The exponent decreases by 1 each time.

The patterns suggest $3^0 = 1$ because $3 \div 3 = 1$.

We can make a similar table for the powers of any integer base except 0.

The Zero Exponent

A power with exponent 0 is equal to 1.

The base of the power can be any integer except 0.

Example 1 Powers with Exponent Zero

Evaluate each expression.

a) 6^0

b) $(-5)^0$

Solution

A power with exponent 0 is equal to 1.

a) $6^0 = 1$

b) $(-5)^0 = 1$

The zero exponent applies to the number in the brackets.

Check

1. Evaluate each expression.

a) $8^0 = \underline{\quad}$

b) $-4^0 = \underline{\quad}$

c) $4^0 = \underline{\quad}$

d) $(-10)^0 = \underline{\quad}$

If there are no brackets, the zero exponent applies only to the base.

Example 2 Powers of Ten

Write as a power of 10.

- a) 10 000 b) 1000 c) 100 d) 10 e) 1

Solution

a) $10\,000 = 10 \times 10 \times 10 \times 10$
 $= 10^4$

b) $1000 = 10 \times 10 \times 10$
 $= 10^3$

c) $100 = 10 \times 10$
 $= 10^2$

d) $10 = 10^1$

e) $1 = 10^0$

Notice that the exponent is equal to the number of zeros.

Check

1. a) $5^1 = \underline{\quad}$

b) $(-7)^1 = \underline{\quad}$

c) $10^1 = \underline{\quad}$

d) $10^0 = \underline{\quad}$

Practice

1. a) Complete the table below.

Power	Repeated Multiplication	Standard Form
5^4	$5 \times 5 \times 5 \times 5$	625
5^3	$5 \times 5 \times 5$	$\underline{\quad}$
5^2	$\underline{\quad}$	$\underline{\quad}$
5^1	$\underline{\quad}$	$\underline{\quad}$

b) What is the value of 5^1 ? $\underline{\quad}$

c) Use the table. What is the value of 5^0 ? $\underline{\quad}$

2. Evaluate each power.

a) $2^0 =$ _____

b) $9^0 =$ _____

c) $(-2)^0 =$ _____

d) $-2^0 =$ _____

e) $10^1 =$ _____

f) $(-8)^1 =$ _____

If there are no brackets, the exponent applies only to the base.

3. Write each number as a power of 10.

a) 10 000 = 10 _____

b) 1 000 000 = 10 _____

c) Ten million = _____

d) One = _____

e) 1 000 000 000 = _____

f) 10 = _____

4. Evaluate each power of 10.

a) $-10^6 =$ _____

b) $-10^0 =$ _____

c) $-10^8 =$ _____

d) $-10^1 =$ _____

5. One trillion is written as 1 000 000 000 000.

Write each number as a power of 10.

a) One trillion = 1 000 000 000 000 = _____

b) Ten trillion = $10 \times$ _____ = _____

c) One hundred trillion = _____ = _____

6. Write each number in standard form.

a) $5 \times 10^4 = 5 \times 10\,000$
= _____

b) $(4 \times 10^2) + (3 \times 10^1) + (7 \times 10^0) = (4 \times 100) +$ _____
= _____
= _____

c) $(2 \times 10^3) + (6 \times 10^2) + (4 \times 10^1) + (9 \times 10^0)$
= _____
= _____
= _____

d) $(7 \times 10^3) + (8 \times 10^0) =$ _____
= _____
= _____


2.3 Skill Builder



Adding Integers

To add a positive integer and a negative integer: $7 + (-4)$

- Model each integer with tiles.
- Circle zero pairs.

7: 
-4: 


There are 4 zero pairs.
There are 3  tiles left.
They model 3.
So, $7 + (-4) = 3$

Each pair of 1  tile and 1  tile makes a zero pair. The pair models 0.

To add 2 negative integers: $(-4) + (-2)$

- Model each integer with tiles.
- Combine the tiles.

-4: 
-2: 

There are 6  tiles.
They model -6 .
So, $(-4) + (-2) = -6$

Check

1. Add.

a) $(-3) + (-4) = \underline{\quad}$

b) $6 + (-2) = \underline{\quad}$

c) $(-5) + 2 = \underline{\quad}$

d) $(-4) + (-4) = \underline{\quad}$

2. a) Kerry borrows \$5. Then she borrows another \$5.

Add to show what Kerry owes.

$(-5) + (-5) = \underline{\quad}$

Kerry owes \$.

When an amount of money is negative, it is owed.

b) The temperature was 8°C . It fell 10°C .

Add to show the new temperature.

$8 + (\underline{\quad}) = \underline{\quad}$

The new temperature is $^{\circ}\text{C}$.

Subtracting Integers

To subtract 2 integers: $3 - 6$

- Model the first integer.
- Take away the number of tiles equal to the second integer.

Model 3.



There are not enough tiles to take away 6.

To take away 6, we need 3 more light gray tiles.

We add zero pairs. Add 3 light gray tiles and 3 dark gray tiles.



Now take away the 6 light gray tiles.



Since 3 dark gray tiles remain, we write: $3 - 6 = -3$

When tiles are not available, think of subtraction as the opposite of addition.

To subtract an integer, add its opposite integer.

For example,

$$(-3) - (+2) = -5$$

┌───┐
↓
Subtract +2.

$$(-3) + (-2) = -5$$

┌───┐
↓
Add -2.

*Adding zero pairs
does not change
the value. Zero
pairs represent 0.*

Check

1. Subtract.

a) $(-6) - 2 = \underline{\quad}$

b) $2 - (-6) = \underline{\quad}$

c) $(-8) - 9 = \underline{\quad}$

d) $8 - (-9) = \underline{\quad}$

Dividing Integers

When dividing 2 integers, look at the sign of each integer:

- When the integers have the same sign, their quotient is positive.
- When the integers have different signs, their quotient is negative.

The same rule applies to the multiplication of integers.

$6 \div (-3)$ These 2 integers have different signs, so their quotient is negative.

$$6 \div (-3) = -2$$

$(-10) \div (-2)$ These 2 integers have the same sign, so their quotient is positive.

$$(-10) \div (-2) = 5$$

Check

1. Calculate.

a) $(-4) \div 2$
= _____

b) $(-6) \div (-3)$
= _____

c) $15 \div (-3)$
= _____

2.3 Order of Operations with Powers

FOCUS Explain and apply the order of operations with exponents.

We use this order of operations when evaluating an expression with powers:

- Do the operations in brackets first.
- Evaluate the powers.
- Multiply and divide, in order, from left to right.
- Add and subtract, in order, from left to right.

We can use the word BEDMAS to help us remember the order of operations:

- B** Brackets
E Exponents
D Division
M Multiplication
A Addition
S Subtraction

Example 1 Adding and Subtracting with Powers

Evaluate.

a) $2^3 + 1$

b) $8 - 3^2$

c) $(3 - 1)^3$

Solution

a) $2^3 + 1$

$$\begin{aligned} &= (2)(2)(2) + 1 \\ &= 8 + 1 \\ &= 9 \end{aligned}$$

Evaluate the power first: 2^3

Multiply: $(2)(2)(2)$

Then add: $8 + 1$

b) $8 - 3^2$

$$\begin{aligned} &= 8 - (3)(3) \\ &= 8 - 9 \\ &= -1 \end{aligned}$$

Evaluate the power first: 3^2

Multiply: $(3)(3)$

Then subtract: $8 - 9$

c) $(3 - 1)^3$

$$\begin{aligned} &= 2^3 \\ &= (2)(2)(2) \\ &= 8 \end{aligned}$$

Subtract inside the brackets first: $3 - 1$

Evaluate the power: 2^3

Multiply: $(2)(2)(2)$

To subtract,
add the
opposite:
 $8 + (-9)$

Check

1. Evaluate.

$$\begin{aligned}\text{a) } 4^2 + 3 &= \underline{\quad\quad} + 3 \\ &= \underline{\quad\quad} \\ &= \underline{\quad\quad}\end{aligned}$$

$$\begin{aligned}\text{b) } 5^2 - 2^2 &= \underline{\quad\quad} - (2)(2) \\ &= \underline{\quad\quad} \\ &= \underline{\quad\quad}\end{aligned}$$

$$\begin{aligned}\text{c) } (2 + 1)^2 &= \underline{\quad\quad}^2 \\ &= \underline{\quad\quad} \\ &= \underline{\quad\quad}\end{aligned}$$

$$\begin{aligned}\text{d) } (5 - 6)^2 &= \underline{\quad\quad} \\ &= \underline{\quad\quad} \\ &= \underline{\quad\quad}\end{aligned}$$

Example 2 Multiplying and Dividing with Powers

Evaluate.

$$\text{a) } [2 \times (-2)^3]^2$$

Curved brackets Square brackets

$$\text{b) } (7^2 + 5^0) \div (-5)^1$$

When we need curved brackets for integers, we use square brackets to show the order of operations.

Solution

$$\begin{aligned}\text{a) } [2 \times (-2)^3]^2 & \\ &= [2 \times (-8)]^2 \\ &= (-16)^2 \\ &= 256\end{aligned}$$

Evaluate what is inside the square brackets first: $2 \times (-2)^3$
Start with $(-2)^3 = -8$.

$$\begin{aligned}\text{b) } (7^2 + 5^0) \div (-5)^1 & \\ &= (49 + 1) \div (-5)^1 \\ &= 50 \div (-5)^1 \\ &= 50 \div (-5) \\ &= -10\end{aligned}$$

Evaluate what is inside the brackets first: $7^2 + 5^0$
Add inside the brackets: $49 + 1$
Evaluate the power: $(-5)^1$

Check

1. Evaluate.

$$\begin{aligned}\text{a) } 5 \times 3^2 &= 5 \times \underline{\quad} \underline{\quad} \\ &= 5 \times \underline{\quad} \\ &= \underline{\quad}\end{aligned}$$

$$\begin{aligned}\text{b) } 8^2 \div 4 &= \underline{\quad} \underline{\quad} \div 4 \\ &= \underline{\quad} \div 4 \\ &= \underline{\quad}\end{aligned}$$

$$\begin{aligned}\text{c) } (3^2 + 6^0)^2 \div 2^1 \\ &= (\underline{\quad} + \underline{\quad})^2 \div 2^1 \\ &= \underline{\quad} \div 2^1 \\ &= \underline{\quad} \div \underline{\quad} \\ &= \underline{\quad}\end{aligned}$$

$$\begin{aligned}\text{d) } 10^2 + (2 \times 2^2)^2 &= 10^2 + (2 \times \underline{\quad})^2 \\ &= 10^2 + \underline{\quad} \\ &= \underline{\quad} + \underline{\quad} \\ &= \underline{\quad}\end{aligned}$$

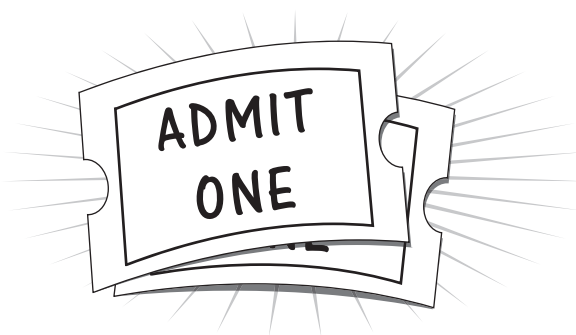
Example 3 Solving Problems Using Powers

Corin answered the following skill-testing question to win free movie tickets:

$$120 + 20^3 \div 10^3 + 12 \times 120$$

His answer was 1568.

Did Corin win the movie tickets? Show your work.



Solution

$$\begin{aligned}120 + 20^3 \div 10^3 + 12 \times 120 \\ &= 120 + 8000 \div 1000 + 12 \times 120 \\ &= 120 + 8 + 1440 \\ &= 1568\end{aligned}$$

Corin won the movie tickets.

Evaluate the powers first: 20^3 and 10^3

Divide and multiply.

Add: $120 + 8 + 1440$

Check

1. Answer the following skill-testing question to enter a draw for a Caribbean cruise.

$$\begin{aligned}(6 + 4) + 3^2 \times 10 - 10^2 \div 4 \\ &= \underline{\quad} \\ &= \underline{\quad} \\ &= \underline{\quad}\end{aligned}$$

Practice

1. Evaluate.

$$\begin{aligned} \text{a) } 2^2 + 1 &= \underline{\quad} + 1 \\ &= \underline{\quad} + 1 \\ &= \underline{\quad} \end{aligned}$$

$$\begin{aligned} \text{b) } 2^2 - 1 &= \underline{\quad} - 1 \\ &= \underline{\quad} - 1 \\ &= \underline{\quad} \end{aligned}$$

$$\begin{aligned} \text{c) } (2 + 1)^2 &= \underline{\quad} \\ &= \underline{\quad} \\ &= \underline{\quad} \end{aligned}$$

$$\begin{aligned} \text{d) } (2 - 1)^2 &= \underline{\quad} \\ &= \underline{\quad} \\ &= \underline{\quad} \end{aligned}$$

2. Evaluate.

$$\begin{aligned} \text{a) } 4 \times 2^2 &= 4 \times \underline{\quad} \\ &= 4 \times \underline{\quad} \\ &= \underline{\quad} \end{aligned}$$

$$\begin{aligned} \text{b) } 4^2 \times 2 &= \underline{\quad} \times 2 \\ &= \underline{\quad} \times 2 \\ &= \underline{\quad} \end{aligned}$$

$$\begin{aligned} \text{c) } (4 \times 2)^2 &= \underline{\quad} \\ &= \underline{\quad} \\ &= \underline{\quad} \end{aligned}$$

$$\begin{aligned} \text{d) } (-4)^2 \div 2 &= \underline{\quad} \div 2 \\ &= \underline{\quad} \div 2 \\ &= \underline{\quad} \end{aligned}$$

3. Evaluate.

$$\begin{aligned} \text{a) } 2^3 + (-1)^3 &= \underline{\quad} + (-1)^3 \\ &= \underline{\quad} + (-1)^3 \\ &= \underline{\quad} + \underline{\quad} \quad \underline{\quad} \quad \underline{\quad} \\ &= \underline{\quad} + \underline{\quad} \\ &= \underline{\quad} \end{aligned}$$

$$\begin{aligned} \text{b) } (2 - 1)^3 &= \underline{\quad} \\ &= \underline{\quad} \\ &= \underline{\quad} \end{aligned}$$

$$\begin{aligned} \text{c) } 2^3 - (-1)^3 &= \underline{\quad} - (-1)^3 \\ &= \underline{\quad} - (-1)^3 \\ &= \underline{\quad} - \underline{\quad} \quad \underline{\quad} \quad \underline{\quad} \\ &= \underline{\quad} - \underline{\quad} \\ &= \underline{\quad} \end{aligned}$$

$$\begin{aligned} \text{d) } (2 + 1)^3 &= \underline{\quad} \\ &= \underline{\quad} \\ &= \underline{\quad} \end{aligned}$$

4. Evaluate.

$$\begin{aligned} \text{a) } 3^2 \div (-1)^2 &= \underline{\quad} \div (-1)^2 \\ &= \underline{\quad} \div (-1)^2 \\ &= \underline{\quad} \div \underline{\quad} \quad \underline{\quad} \quad \underline{\quad} \\ &= \underline{\quad} \div \underline{\quad} \\ &= \underline{\quad} \end{aligned}$$

$$\begin{aligned} \text{b) } (3 \div 1)^2 &= \underline{\quad}^2 \\ &= \underline{\quad} \\ &= \underline{\quad} \end{aligned}$$

$$\begin{aligned} \text{c) } 3^2 \times (-2)^2 &= \underline{\quad} \times (-2)^2 \\ &= \underline{\quad} \times (-2)^2 \\ &= \underline{\quad} \times \underline{\quad} \\ &= \underline{\quad} \times \underline{\quad} \\ &= \underline{\quad} \end{aligned}$$

$$\begin{aligned} \text{d) } 5^2 \div (-5)^1 &= \underline{\quad} \div (-5)^1 \\ &= \underline{\quad} \div (-5)^1 \\ &= \underline{\quad} \div \underline{\quad} \\ &= \underline{\quad} \end{aligned}$$

5. Evaluate.

a) $(-2)^0 \times (-2) = \underline{\quad} \times (-2)$
 $= \underline{\quad}$

b) $2^3 \div (-2)^2 = \underline{\quad} \div (-2)^2$
 $= \underline{\quad} \div (-2)^2$
 $= \underline{\quad} \div \underline{\quad}$
 $= \underline{\quad} \div \underline{\quad}$
 $= \underline{\quad}$

c) $(3 + 2)^0 + (3 \times 2)^0 = \underline{\quad} + \underline{\quad}$
 $= \underline{\quad}$

d) $(3 \times 5^2)^0 = \underline{\quad}$

e) $(2)(3) - (4)^2 = (2)(3) - \underline{\quad}$
 $= (2)(3) - \underline{\quad}$
 $= \underline{\quad} - \underline{\quad}$
 $= \underline{\quad}$

f) $3(2 - 1)^2 = 3 \underline{\quad}$
 $= 3 \underline{\quad}$
 $= \underline{\quad}$

A power with exponent 0 is equal to 1.

g) $(-2)^2 + (3)(4) = \underline{\quad} + (3)(4)$
 $= \underline{\quad} + (3)(4)$
 $= \underline{\quad} + \underline{\quad}$
 $= \underline{\quad}$

h) $(-2) + 3^0 \times (-2) = (-2) + \underline{\quad} \times (-2)$
 $= (-2) + \underline{\quad}$
 $= \underline{\quad}$

6. Amaya wants to replace the hardwood floor in her house.

Here is how she calculates the cost, in dollars:

$$70 \times 6^2 + 60 \times 6^2$$

How much will it cost Amaya to replace the hardwood floor?

$$70 \times \underline{\quad} + 60 \times \underline{\quad}$$

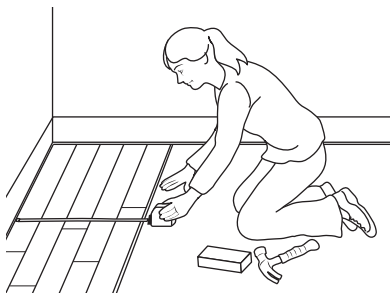
$$= 70 \times \underline{\quad} + 60 \times \underline{\quad}$$

$$= \underline{\quad} + \underline{\quad}$$

$$= \underline{\quad}$$

It will cost Amaya \$ $\underline{\quad}$ to replace the hardwood floor.

Remember the order of operations: BEDMAS





Can you ...

- Use powers to show repeated multiplication?
- Use patterns to evaluate a power with exponent zero, such as 50^0 ?
- Use the correct order of operations with powers?

2.1 1. Give the base and exponent of each power.

- a) 6^2 Base: _____ Exponent: _____
There are _____ factors of _____.
- b) 4^5 Base: _____ Exponent: _____
There are _____ factors of _____.
- c) $(-3)^8$ Base: _____ Exponent: _____
There are _____ factors of _____.
- d) -3^8 Base: _____ Exponent: _____
There are _____ factors of _____.

2. Write as a power.

- a) $7 \times 7 \times 7 \times 7 \times 7 \times 7 = 7^{\text{---}}$
- b) $2 \times 2 \times 2 \times 2 = 2^{\text{---}}$
- c) $5 = \text{---}$
- d) $(-5)(-5)(-5)(-5)(-5) = \text{---}$

3. Write each power as repeated multiplication and in standard form.

- a) $5^2 = 5 \times \text{---} = \text{---}$
- b) $2^3 = \text{---} = \text{---}$
- c) $3^4 = \text{---} = \text{---}$

2.2 4. a) Complete the table.

Power	Repeated Multiplication	Standard Form
7^3	$7 \times 7 \times 7$	343
7^2	7×7	
7^1		

b) What is the value of 7^0 ? _____

5. Write each number in standard form and as a power of 10.

a) One hundred = 100
= 10 _____

b) Ten thousand = _____
= 10 _____

c) One million = _____
= 10 _____

d) One = _____
= 10 _____

6. Evaluate.

a) $6^0 =$ _____

b) $(-8)^0 =$ _____

c) $12^1 =$ _____

d) $-8^0 =$ _____

7. Write each number in standard form.

a) 4×10^3
= $4 \times$ _____
= _____

b) $(1 \times 10^3) + (3 \times 10^2) + (2 \times 10^1) + (1 \times 10^0)$
= $(1 \times 1000) + (3 \times \text{_____}) + (\text{_____}) + (\text{_____})$
= _____ + _____ + _____ + _____
= _____

c) $(4 \times 10^3) + (2 \times 10^2) + (3 \times 10^1) + (6 \times 10^0)$
= $(4 \times \text{_____}) + (\text{_____}) + (\text{_____}) + (\text{_____})$
= _____ + _____ + _____ + _____
= _____

d) $(8 \times 10^2) + (1 \times 10^1) + (9 \times 10^0)$
= _____ + _____ + _____
= _____
= _____

2.3 8. Evaluate.

$$\begin{aligned} \text{a) } 3^2 + 5 &= \underline{\quad\quad} + 5 \\ &= \underline{\quad\quad} + 5 \\ &= \underline{\quad\quad} \end{aligned}$$

$$\begin{aligned} \text{b) } 5^2 - 2^3 &= \underline{\quad\quad} - 2^3 \\ &= \underline{\quad\quad} - 2^3 \\ &= \underline{\quad\quad} - \underline{\quad\quad\quad} \\ &= \underline{\quad\quad\quad} \\ &= \underline{\quad\quad} \end{aligned}$$

$$\begin{aligned} \text{c) } (2 + 3)^3 &= (\underline{\quad})^3 \\ &= \underline{\quad\quad\quad\quad\quad} \\ &= \underline{\quad\quad} \end{aligned}$$

$$\begin{aligned} \text{d) } 2^3 + (-3)^3 &= \underline{\quad\quad\quad} + (-3)^3 \\ &= \underline{\quad\quad} + (-3)^3 \\ &= \underline{\quad\quad} + \underline{\quad\quad\quad\quad\quad} \\ &= \underline{\quad\quad\quad\quad\quad} \\ &= \underline{\quad\quad} \end{aligned}$$

9. Evaluate.

$$\begin{aligned} \text{a) } 5 \times 3^2 &= 5 \times \underline{\quad\quad} \\ &= \underline{\quad\quad} \end{aligned}$$

$$\begin{aligned} \text{b) } 8^2 \div 4 &= \underline{\quad\quad} \div 4 \\ &= \underline{\quad\quad} \end{aligned}$$

$$\begin{aligned} \text{c) } (10 + 2) \div 2^2 &= \underline{\quad\quad} \div 2^2 \\ &= \underline{\quad\quad} \div \underline{\quad\quad} \\ &= \underline{\quad\quad} \end{aligned}$$

$$\begin{aligned} \text{d) } (7^2 + 1) \div (2^3 + 2) &= (\underline{\quad\quad} + 1) \div (\underline{\quad\quad} + 2) \\ &= \underline{\quad\quad} \div \underline{\quad\quad} \\ &= \underline{\quad\quad} \end{aligned}$$

10. Evaluate. State which operation you do first.

$$\begin{aligned} \text{a) } 3^2 + 4^2 &\underline{\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad} \\ &= \underline{\quad\quad} + \underline{\quad\quad} \\ &= \underline{\quad\quad} + \underline{\quad\quad} \\ &= \underline{\quad\quad} \end{aligned}$$

$$\begin{aligned} \text{b) } [(-3) - 2]^3 &\underline{\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad} \\ &= (\underline{\quad\quad})^3 \\ &= \underline{\quad\quad\quad\quad\quad} \\ &= \underline{\quad\quad} \end{aligned}$$

$$\begin{aligned} \text{c) } (-2)^3 + (-3)^0 &\underline{\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad} \\ &= \underline{\quad\quad\quad\quad\quad} + \underline{\quad\quad} \\ &= \underline{\quad\quad\quad\quad\quad} \\ &= \underline{\quad\quad} \end{aligned}$$

$$\begin{aligned} \text{d) } [(6 - 3)^3 \times (2 + 2)^2]^0 &\underline{\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad} \\ &= \underline{\quad\quad} \end{aligned}$$

2.4 Skill Builder

Simplifying Fractions

To simplify a fraction, divide the numerator and denominator by their common factors.

To simplify $\frac{5 \times 5 \times 5 \times 5}{5 \times 5}$:

This fraction shows repeated multiplication.

Divide the numerator and denominator by their common factors: 5×5 .

$$\begin{aligned} & \frac{\cancel{5}^1 \times \cancel{5}^1 \times 5 \times 5}{\cancel{5}^1 \times \cancel{5}^1} \\ &= \frac{5 \times 5}{1} \\ &= 25 \end{aligned}$$

Check

1. Simplify each fraction.

a) $\frac{3 \times 3 \times 3}{3}$

= _____
= _____

b) $\frac{8 \times 8 \times 8 \times 8 \times 8}{8 \times 8 \times 8 \times 8 \times 8}$

= _____

c) $\frac{5 \times 5 \times 5 \times 5 \times 5}{5 \times 5 \times 5}$

= _____
= _____

d) $\frac{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2}$

= _____
= _____

What are the common factors?

2.4 Exponent Laws I

FOCUS Understand and apply the exponent laws for products and quotients of powers.

Multiply $3^2 \times 3^4$.

$3^2 \times 3^4$ Write as repeated multiplication.

$$= \underbrace{(3 \times 3)}_{\text{2 factors of 3}} \times \underbrace{(3 \times 3 \times 3 \times 3)}_{\text{4 factors of 3}}$$

$$= \underbrace{3 \times 3 \times 3 \times 3 \times 3 \times 3}_{\text{6 factors of 3}}$$

$$= 3^6$$

↑ Base ↘ Exponent

So, $3^2 \times 3^4 = 3^6$ Look at the pattern in the exponents.

$$\begin{array}{ccc} 3^2 & \times & 3^4 & = & 3^6 \\ \downarrow & & \downarrow & & \downarrow \\ 2 & + & 4 & = & 6 \end{array}$$

$$\begin{aligned} \text{We write: } 3^2 \times 3^4 &= 3^{(2+4)} \\ &= 3^6 \end{aligned}$$

This relationship is true when you multiply any 2 powers with the same base.

Exponent Law for a Product of Powers

To multiply powers with the same base, add the exponents.

Example 1 Simplifying Products with the Same Base

Write as a power.

a) $5^3 \times 5^4$

b) $(-6)^2 \times (-6)^3$

c) $(7^2)(7)$

Solution

a) The powers have the same base: 5

Use the exponent law for products: add the exponents.

$$\begin{aligned} 5^3 \times 5^4 &= 5^{(3+4)} \\ &= 5^7 \end{aligned}$$

*To check your work,
you can write the
powers as repeated
multiplication.*

b) The powers have the same base: -6

$$\begin{aligned}(-6)^2 \times (-6)^3 &= (-6)^{(2+3)} \quad \text{Add the exponents.} \\ &= (-6)^5\end{aligned}$$

c) $(7^2)(7) = 7^2 \times 7^1$
 $= 7^{(2+1)}$
 $= 7^3$

Use the exponent law for products.
Add the exponents.

*7 can be written
as 7^1 .*

Check

1. Write as a power.

a) $2^5 \times 2^4 = 2^{(\underline{\quad} + \underline{\quad})}$
 $= 2^{\underline{\quad}}$

b) $5^2 \times 5^5 = 5^{\underline{\quad}}$
 $= 5^{\underline{\quad}}$

c) $(-3)^2 \times (-3)^3 = \underline{\hspace{2cm}}$
 $= \underline{\hspace{2cm}}$

d) $10^5 \times 10 = \underline{\hspace{2cm}}$
 $= \underline{\hspace{2cm}}$

Divide $3^4 \div 3^2$.

$$3^4 \div 3^2 = \frac{3^4}{3^2}$$

$$= \frac{3 \times 3 \times 3 \times 3}{3 \times 3}$$

$$= \frac{\cancel{3}^1 \times \cancel{3}^1 \times 3 \times 3}{\cancel{3}^1 \times \cancel{3}^1}$$

$$= \frac{3 \times 3}{1}$$

$$= 3 \times 3$$

$$= 3^2$$

So, $3^4 \div 3^2 = 3^2$
 $\downarrow \quad \downarrow \quad \downarrow$
 $4 - 2 = 2$

Simplify.

*We can show division
in fraction form.*

Look at the pattern in the exponents.

We write: $3^4 \div 3^2 = 3^{(4-2)}$
 $= 3^2$

This relationship is true when you divide any 2 powers with the same base.

Exponent Law for a Quotient of Powers

To divide powers with the same base, subtract the exponents.

Example 2 Simplifying Quotients with the Same Base

Write as a power.

a) $4^5 \div 4^3$

b) $(-2)^7 \div (-2)^2$

Solution

Use the exponent law for quotients: subtract the exponents.

a) $4^5 \div 4^3 = 4^{(5-3)}$
 $= 4^2$

The powers have the same base: 4

b) $(-2)^7 \div (-2)^2 = (-2)^{(7-2)}$
 $= (-2)^5$

To check your work, you can write the powers as repeated multiplication.

The powers have the same base: -2

Check

1. Write as a power.

a) $(-5)^6 \div (-5)^3 = (-5)\underline{\hspace{1cm}}$
 $= \underline{\hspace{1cm}}$

b) $\frac{(-3)^9}{(-3)^5} = (-3)\underline{\hspace{1cm}}$
 $= \underline{\hspace{1cm}}$

c) $8^4 \div 8^3 = \underline{\hspace{1cm}}$
 $= \underline{\hspace{1cm}}$

d) $9^8 \div 9^2 = \underline{\hspace{1cm}}$
 $= \underline{\hspace{1cm}}$

$\frac{(-3)^9}{(-3)^5}$ is the same as
 $(-3)^9 \div (-3)^5$

Example 3 Evaluating Expressions Using Exponent Laws

Evaluate.

a) $2^2 \times 2^3 \div 2^4$

b) $(-2)^5 \div (-2)^3 \times (-2)$

Solution

a) $2^2 \times 2^3 \div 2^4$
 $= 2^{(2+3)} \div 2^4$
 $= 2^5 \div 2^4$
 $= 2^{(5-4)}$
 $= 2^1$
 $= 2$

Add the exponents of the 2 powers that are multiplied.
Then, subtract the exponent of the power that is divided.

b) $(-2)^5 \div (-2)^3 \times (-2)$
 $= (-2)^{(5-3)} \times (-2)$
 $= (-2)^2 \times (-2)$
 $= (-2)^{(2+1)}$
 $= (-2)^3$
 $= (-2)(-2)(-2)$
 $= -8$

Subtract the exponents of the 2 powers that are divided.

Multiply: add the exponents.

Check

1. Evaluate.

a) $4 \times 4^3 \div 4^2 = 4(\underline{\quad} + \underline{\quad}) \div 4^2$
 $= 4\underline{\quad} \div 4^2$
 $= 4(\underline{\quad} - \underline{\quad})$
 $= 4\underline{\quad}$
 $= \underline{\quad}$

b) $(-3) \div (-3) \times (-3)$
 $= (-3)\underline{\quad} \times (-3)$
 $= (-3)\underline{\quad} \times (-3)$
 $= (-3)\underline{\quad}$
 $= (-3)\underline{\quad}$
 $= \underline{\quad}$

$(-3) = (-3)^1$

Practice

1. Write each product as a single power.

a) $7^6 \times 7^2 = 7(\underline{\quad} + \underline{\quad})$
 $= 7\underline{\quad}$

b) $(-4)^5 \times (-4)^3 = (-4)\underline{\quad}$
 $= (-4)\underline{\quad}$

c) $(-2) \times (-2)^3 = \underline{\quad}$
 $= \underline{\quad}$

d) $10^5 \times 10^5 = \underline{\quad}$
 $= \underline{\quad}$

e) $7^0 \times 7^1 = \underline{\quad}$
 $= \underline{\quad}$

f) $(-3)^4 \times (-3)^5 = \underline{\quad}$
 $= \underline{\quad}$

To multiply powers with the same base, add the exponents.

2. Write each quotient as a power.

a) $(-3)^5 \div (-3)^2 = (-3)(\underline{\quad} - \underline{\quad})$
 $= (-3)\underline{\quad}$

b) $5^6 \div 5^4 = 5\underline{\quad}$
 $= 5\underline{\quad}$

c) $\frac{4^7}{4^4} = 4\underline{\quad}$
 $= 4\underline{\quad}$

d) $\frac{5^8}{5^6} = \underline{\quad}$
 $= \underline{\quad}$

e) $6^4 \div 6^4 = \underline{\quad}$
 $= \underline{\quad}$

f) $\frac{(-6)^8}{(-6)^7} = \underline{\quad}$
 $= \underline{\quad}$

To divide powers with the same base, subtract the exponents.

3. Write as a single power.

a) $2^3 \times 2^4 \times 2^5 = 2(\underline{\quad} + \underline{\quad}) \times 2^5$
 $= 2\underline{\quad} \times 2^5$
 $= 2\underline{\quad}$
 $= 2\underline{\quad}$

b) $\frac{3^2 \times 3^2}{3^2 \times 3^2} = \frac{3\underline{\quad}}{3\underline{\quad}}$
 $= \frac{3\underline{\quad}}{3\underline{\quad}}$
 $= \underline{\quad}$
 $= \underline{\quad}$

Which exponent law should you use?

c) $10^3 \times 10^5 \div 10^2 = \underline{\quad} \div 10^2$
 $= \underline{\quad} \div 10^2$
 $= \underline{\quad}$
 $= \underline{\quad}$

d) $(-1)^9 \div (-1)^5 \times (-1)^0$
 $= \underline{\quad} \times (-1)^0$
 $= \underline{\quad} \times (-1)^0$
 $= \underline{\quad}$
 $= \underline{\quad}$

4. Simplify, then evaluate.

$$\begin{aligned} \text{a) } & (-3)^1 \times (-3)^2 \times 2 \\ &= \underline{\hspace{2cm}} \times 2 \\ &= \underline{\hspace{2cm}} \times 2 \\ &= \underline{\hspace{2cm}} \times 2 \\ &= \underline{\hspace{2cm}} \end{aligned}$$

$$\begin{aligned} \text{b) } & 9^9 \div 9^7 \times 9^0 = \underline{\hspace{2cm}} \times 9^0 \\ &= \underline{\hspace{2cm}} \times 9^0 \\ &= \underline{\hspace{2cm}} \\ &= \underline{\hspace{2cm}} \\ &= \underline{\hspace{2cm}} \end{aligned}$$

See if you can use the exponent laws to simplify.

$$\begin{aligned} \text{c) } & \frac{5^2}{5^0} = \underline{\hspace{2cm}} \\ &= \underline{\hspace{2cm}} \\ &= \underline{\hspace{2cm}} \end{aligned}$$

$$\begin{aligned} \text{d) } & \frac{5^5}{5^4} \times 5 = 5 \underline{\hspace{1cm}} \times 5 \\ &= 5 \underline{\hspace{1cm}} \times 5 \\ &= 5 \underline{\hspace{1cm}} \\ &= 5 \underline{\hspace{1cm}} \\ &= \underline{\hspace{2cm}} \end{aligned}$$

5. Identify any errors and correct them.

a) $4^3 \times 4^5 = 4^8$ _____

b) $2^5 \times 2^5 = 2^{25}$ _____

c) $(-3)^6 \div (-3)^2 = (-3)^3$ _____

d) $7^0 \times 7^2 = 7^0$ _____

e) $6^2 + 6^2 = 6^4$ _____

f) $10^6 \div 10 = 10^6$ _____

g) $2^3 \times 5^2 = 10^5$ _____

2.5 Skill Builder

Grouping Equal Factors

In multiplication, you can group equal factors.

For example:

$$\begin{aligned}
 &3 \times 7 \times 3 \times 7 \times 7 \times 3 \\
 &= \underbrace{3 \times 3 \times 3} \times \underbrace{7 \times 7 \times 7} \\
 &= 3^3 \times 7^3
 \end{aligned}$$

Group equal factors.

Write repeated multiplication as powers.

Order does not matter in multiplication.

Check

1. Group equal factors and write as powers.

a) $2 \times 10 \times 2 \times 10 \times 2 = \underline{2 \times 2 \times 2 \times \underline{\hspace{2cm}}}$
 $= \underline{\hspace{2cm}}$

b) $2 \times 5 \times 2 \times 5 \times 2 \times 5 \times 2 \times 5 = \underline{\hspace{2cm}}$
 $= \underline{\hspace{2cm}}$

Multiplying Fractions

To multiply fractions, first multiply the numerators, and then multiply the denominators.

$$\begin{aligned}
 \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} &= \frac{2 \times 2 \times 2 \times 2}{3 \times 3 \times 3 \times 3} \\
 &= \frac{2^4}{3^4}
 \end{aligned}$$

Write repeated multiplication as powers.

There are 4 factors of 2, and 4 factors of 3.

Check

1. Multiply the fractions. Write as powers.

a) $\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \underline{\hspace{2cm}}$
 $= \underline{\hspace{2cm}}$

b) $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$
 $= \underline{\hspace{2cm}}$
 $= \underline{\hspace{2cm}}$

2.5 Exponent Laws II

FOCUS Understand and apply exponent laws for powers of: products; quotients; and powers.

Multiply $3^2 \times 3^2 \times 3^2$.

$$\begin{aligned} 3^2 \times 3^2 \times 3^2 &= 3^{2+2+2} \\ &= 3^6 \end{aligned}$$

Use the exponent law for the product of powers.

Add the exponents.

We can write repeated multiplication as powers.

So, $\underbrace{3^2 \times 3^2 \times 3^2}_{3 \text{ factors of } (3^2)}$

$$= (3^2)^3$$

$$= 3^6$$

$$2 \times 3 = 6$$

We write: $(3^2)^3 = 3^2 \times 3$
 $= 3^6$

The base is 3^2 .

The exponent is 3.

This is a **power of a power**.

Look at the pattern in the exponents.

This is also a power.

Exponent Law for a Power of a Power

To raise a power to a power, multiply the exponents.

For example: $(2^3)^5 = 2^{3 \times 5}$

Example 1 Simplifying a Power of a Power

Write as a power.

a) $(3^2)^4$

b) $[(-5)^3]^2$

c) $-(2^3)^4$

Solution

Use the exponent law for a power of a power: multiply the exponents.

a) $(3^2)^4 = 3^{2 \times 4}$
 $= 3^8$

b) $[(-5)^3]^2 = (-5)^{3 \times 2}$ The base is -5 .
 $= (-5)^6$

c) $-(2^3)^4 = -(2^{3 \times 4})$ The base is 2.
 $= -2^{12}$

Check

1. Write as a power.

$$\begin{aligned} \text{a) } (9^3)^4 &= 9 __ \times __ \\ &= 9 __ \end{aligned}$$

$$\begin{aligned} \text{b) } [(-2)^5]^3 &= (-2) __ \\ &= (-2) __ \end{aligned}$$


$$\begin{aligned} \text{c) } -(5^4)^2 &= -(5 __) \\ &= -5 __ \end{aligned}$$

Multiply $(3 \times 4)^2$.

Write as repeated multiplication.

$$\begin{aligned} (3 \times 4)^2 &= (3 \times 4) \times (3 \times 4) \\ &= 3 \times 4 \times 3 \times 4 \\ &= \underbrace{(3 \times 3)} \times \underbrace{(4 \times 4)} \\ &\quad \text{2 factors of 3} \quad \text{2 factors of 4} \\ &= 3^2 \times 4^2 \end{aligned}$$

So, $(3 \times 4)^2 = 3^2 \times 4^2$



power product power

The base of the power is a product: $\underbrace{3 \times 4}_{\text{base}}$

Remove the brackets.

Group equal factors.

Write as powers.

Exponent Law for a Power of a Product

The power of a product is the product of powers.

For example: $(2 \times 3)^4 = 2^4 \times 3^4$

Example 2 Evaluating Powers of Products

Evaluate.

$$\text{a) } (2 \times 5)^2$$

$$\text{b) } [(-3) \times 4]^2$$

Solution

Use the exponent law for a power of a product.

$$\begin{aligned} \text{a) } (2 \times 5)^2 &= 2^2 \times 5^2 \\ &= (2)(2) \times (5)(5) \\ &= 4 \times 25 \\ &= 100 \end{aligned}$$

$$\begin{aligned} \text{b) } [(-3) \times 4]^2 &= (-3)^2 \times 4^2 \\ &= (-3)(-3) \times (4)(4) \\ &= 9 \times 16 \\ &= 144 \end{aligned}$$

Or, use the order of operations and evaluate what is inside the brackets first.

$$\begin{aligned} \text{a) } (2 \times 5)^2 &= 10^2 \\ &= 100 \end{aligned}$$

$$\begin{aligned} \text{b) } [(-3) \times 4]^2 &= (-12)^2 \\ &= 144 \end{aligned}$$

Check

1. Write as a product of powers.

a) $(5 \times 7)^4 = \underline{\quad} \times \underline{\quad}$

b) $(8 \times 2)^2 = \underline{\quad} \times \underline{\quad}$

2. Evaluate.

a) $[(-1) \times 6]^2 = \underline{\quad}^2$
 $= \underline{\quad}$

b) $[(-1) \times (-4)]^3 = \underline{\quad}^3$
 $= \underline{\quad}$

Evaluate $\left(\frac{3}{4}\right)^2$.
base

The base of the power is a quotient: $\frac{3}{4}$

Write as repeated multiplication.

$$\begin{aligned}\left(\frac{3}{4}\right)^2 &= \left(\frac{3}{4}\right) \times \left(\frac{3}{4}\right) \\ &= \frac{3}{4} \times \frac{3}{4} \\ &= \frac{3 \times 3}{4 \times 4} \\ &= \frac{3^2}{4^2}\end{aligned}$$

Multiply the fractions.

Write repeated multiplication as powers.

So, $\left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2}$

power
quotient
power

Exponent Law for a Power of a Quotient

The power of a quotient is the quotient of powers.

For example: $\left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4}$

Example 3 Evaluating Powers of Quotients

Evaluate.

a) $[30 \div (-5)]^2$

b) $\left(\frac{20}{4}\right)^2$

Solution

Use the exponent law for a power of a quotient.

$$\begin{aligned} \text{a) } [30 \div (-5)]^2 &= \left(\frac{30}{-5}\right)^2 & \text{b) } \left(\frac{20}{4}\right)^2 &= \frac{20^2}{4^2} \\ &= \frac{30^2}{(-5)^2} & &= \frac{400}{16} \\ &= \frac{900}{25} & &= 25 \\ &= 36 \end{aligned}$$

Or, use the order of operations and evaluate what is inside the brackets first.

$$\begin{aligned} \text{a) } [30 \div (-5)]^2 &= (-6)^2 & \text{b) } \left(\frac{20}{4}\right)^2 &= 5^2 \\ &= 36 & &= 25 \end{aligned}$$

Check

1. Write as a quotient of powers.

$$\text{a) } \left(\frac{3}{4}\right)^5 = \underline{\hspace{2cm}} \qquad \text{b) } [1 \div (-10)]^3 = \underline{\hspace{2cm}}$$

2. Evaluate.

$$\begin{aligned} \text{a) } [(-16) \div (-4)]^2 & & \text{b) } \left(\frac{36}{6}\right)^3 &= \underline{\hspace{2cm}} \\ = \underline{\hspace{2cm}}^2 & = \underline{\hspace{2cm}} & &= \underline{\hspace{2cm}} \end{aligned}$$

You can evaluate what is inside the brackets first.

Practice

1. Write as a product of powers.

$$\begin{aligned} \text{a) } (5 \times 2)^4 &= 5 \underline{\hspace{1cm}} \times 2 \underline{\hspace{1cm}} & \text{b) } (12 \times 13)^2 &= \underline{\hspace{2cm}} \\ \text{c) } [3 \times (-2)]^3 &= \underline{\hspace{2cm}} & \text{d) } [(-4) \times (-5)]^5 &= \underline{\hspace{2cm}} \end{aligned}$$

2. Write as a quotient of powers.

$$\begin{aligned} \text{a) } (5 \div 8)^0 &= \underline{\hspace{2cm}} & \text{b) } [(-6) \div 5]^7 &= \underline{\hspace{2cm}} \\ \text{c) } \left(\frac{3}{5}\right)^2 &= \underline{\hspace{2cm}} & \text{d) } \left(\frac{-1}{-2}\right)^3 &= \underline{\hspace{2cm}} \end{aligned}$$

3. Write as a power.

a) $(5^2)^3 = 5 \underline{\hspace{1cm}} \times \underline{\hspace{1cm}}$
 $= 5 \underline{\hspace{1cm}}$

b) $[(-2)^3]^5 = (-2) \underline{\hspace{1cm}}$
 $= \underline{\hspace{1cm}}$

c) $(4^4)^1 = \underline{\hspace{1cm}}$
 $= \underline{\hspace{1cm}}$

d) $(8^0)^3 = \underline{\hspace{1cm}}$
 $= \underline{\hspace{1cm}}$

4. Evaluate.

a) $[(6 \times (-2))]^2 = \underline{\hspace{1cm}}$
 $= \underline{\hspace{1cm}}$

b) $-(3 \times 4)^2 = -(\underline{\hspace{1cm}}) \underline{\hspace{1cm}}$
 $= \underline{\hspace{1cm}}$

c) $\left(\frac{-8}{-2}\right)^2 = \underline{\hspace{1cm}}$
 $= \underline{\hspace{1cm}}$

d) $(10 \times 3)^1 = \underline{\hspace{1cm}}$
 $= \underline{\hspace{1cm}}$

e) $[(-2)^1]^2 = \underline{\hspace{1cm}}$
 $= \underline{\hspace{1cm}}$
 $= \underline{\hspace{1cm}}$

f) $[(-2)^1]^3 = \underline{\hspace{1cm}}$
 $= \underline{\hspace{1cm}}$
 $= \underline{\hspace{1cm}}$

5. Find any errors and correct them.

a) $(3^2)^3 = 3^5$ _____

b) $(3 + 2)^2 = 3^2 + 2^2$ _____

c) $(5^3)^3 = 5^9$ _____

d) $\left(\frac{2}{3}\right)^8 = \frac{2^8}{3^8}$ _____

e) $(3 \times 2)^2 = 36$ _____

f) $\left(\frac{2}{3}\right)^2 = \frac{4}{6}$ _____

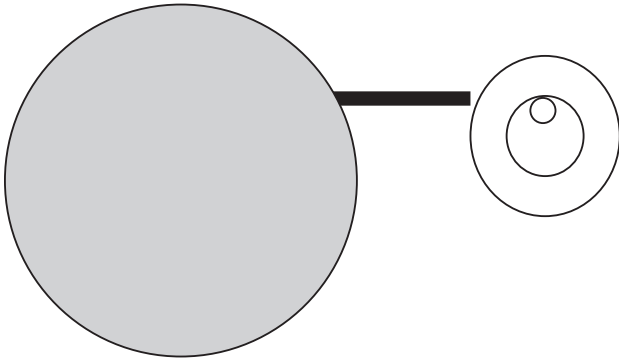
g) $[(-3)^3]^0 = (-3)^3$ _____

h) $[(-2) \times (-3)]^4 = -6^4$ _____

Unit 2 Puzzle

Bird's Eye View

This is a view through the eyes of a bird. What does the bird see?



To find out, simplify or evaluate each expression on the left, then find the answer on the right. Write the corresponding letter beside the question number.

The numbers at the bottom of the page are question numbers.

Write the corresponding letter over each number.

- | | | | |
|---|-------|---|---------|
| 1. $5 \times 5 \times 5 \times 5$ | _____ | A | 100 000 |
| 2. 2^3 | _____ | P | 5^6 |
| 3. $\frac{3^6}{3^2}$ | _____ | S | 0 |
| 4. $4 \times 4 \times 4 \times 4 \times 4$ | _____ | E | 1 |
| 5. $(-2)^3$ | _____ | F | 3^4 |
| 6. $(-2) + 4 \div 2$ | _____ | G | 6 |
| 7. $(5^2)^3$ | _____ | I | 8 |
| 8. $3^2 - 2^3$ | _____ | O | 4^6 |
| 9. $10^2 \times 10^3$ | _____ | N | 4^5 |
| 10. $5 + 3^0$ | _____ | R | 5^4 |
| 11. $4^7 \div 4$ | _____ | Y | -8 |

9 7 8 1 6 11 4 3 1 5 2 4 10 9 4 8 10 10

Unit 2 Study Guide

Skill	Description	Example
Evaluate a power with an integer base.	Write the power as repeated multiplication, then evaluate.	$(-2)^3 = (-2) \times (-2) \times (-2)$ $= -8$
Evaluate a power with an exponent 0.	A power with an integer base and an exponent 0 is equal to 1.	$8^0 = 1$
Use the order of operations to evaluate expressions containing exponents.	Evaluate what is inside the brackets. Evaluate powers. Multiply and divide, in order, from left to right. Add and subtract, in order, from left to right.	$(3^2 + 2) \times (-5)$ $= (9 + 2) \times (-5)$ $= (11) \times (-5)$ $= -55$
Apply the exponent law for a product of powers.	To multiply powers with the same base, add the exponents.	$4^3 \times 4^6 = 4^{3+6}$ $= 4^9$
Apply the exponent law for a quotient of powers.	To divide powers with the same base, subtract the exponents.	$2^7 \div 2^4 = \frac{2^7}{2^4}$ $= 2^{7-4}$ $= 2^3$
Apply the exponent law for a power of a power.	To raise a power to a power, multiply the exponents.	$(5^3)^2 = 5^{3 \times 2}$ $= 5^6$
Apply the exponent law for a power of a product.	Write the power of a product as a product of powers.	$(6 \times 3)^5 = 6^5 \times 3^5$
Apply the exponent law for a power of a quotient.	Write the power of a quotient as a quotient of powers.	$\left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2}$

Unit 2 Review

2.1 1. Give the base and exponent of each power.

a) 6^2 Base _____ Exponent _____

b) $(-3)^8$ Base _____ Exponent _____

2. Write as a power.

a) $4 \times 4 \times 4 = 4^{\text{---}}$

b) $(-3)(-3)(-3)(-3)(-3) = \text{---}$

3. Write each power as repeated multiplication and in standard form.

a) $(-2)^5 = \text{---}$
= _____

b) $10^4 = \text{---}$
= _____

c) Six squared = _____
= _____
= _____

d) Five cubed = _____
= _____
= _____

2.2 4. Evaluate.

a) $10^0 = \text{---}$

b) $(-4)^0 = \text{---}$

c) $8^1 = \text{---}$

d) $-4^0 = \text{---}$

5. Write each number in standard form.

a) 9×10^3
= $9 \times \text{---} \times \text{---} \times \text{---}$
= $9 \times \text{---}$
= _____

$$\begin{aligned} \mathbf{b)} \quad & (1 \times 10^2) + (3 \times 10^1) + (5 \times 10^0) \\ & = (1 \times \underline{\quad}) + (3 \times \underline{\quad}) + (5 \times \underline{\quad}) \\ & = \underline{\hspace{2cm}} \\ & = \underline{\hspace{2cm}} \end{aligned}$$

$$\begin{aligned} \mathbf{c)} \quad & (2 \times 10^3) + (4 \times 10^2) + (1 \times 10^1) + (9 \times 10^0) \\ & = (2 \times \underline{\quad}) + (4 \times \underline{\quad}) + (1 \times \underline{\quad}) + (9 \times \underline{\quad}) \\ & = \underline{\hspace{2cm}} \\ & = \underline{\hspace{2cm}} \end{aligned}$$

$$\begin{aligned} \mathbf{d)} \quad & (5 \times 10^4) + (3 \times 10^2) + (7 \times 10^1) + (2 \times 10^0) \\ & = \underline{\hspace{2cm}} \\ & = \underline{\hspace{2cm}} \\ & = \underline{\hspace{2cm}} \end{aligned}$$

2.3 6. Evaluate.

$$\begin{aligned} \mathbf{a)} \quad & 3^2 + 3 \\ & = \underline{\quad} + 3 \\ & = \underline{\quad} + 3 \\ & = \underline{\quad} \end{aligned}$$

$$\begin{aligned} \mathbf{b)} \quad & [(-2) + 4]^3 \\ & = \underline{\quad}^3 \\ & = \underline{\hspace{2cm}} \\ & = \underline{\quad} \end{aligned}$$

$$\begin{aligned} \mathbf{c)} \quad & (20 + 5) \div 5^2 = \underline{\quad} \div 5^2 \\ & = \underline{\quad} \div \underline{\quad} \\ & = \underline{\quad} \end{aligned}$$

$$\begin{aligned} \mathbf{d)} \quad & (8^2 - 4) \div (6^2 - 6) \\ & = (\underline{\quad} - 4) \div (\underline{\quad} - 6) \\ & = \underline{\quad} \div \underline{\quad} \\ & = \underline{\quad} \end{aligned}$$

7. Evaluate.

$$\begin{aligned} \mathbf{a)} \quad & 5 \times 3^2 = 5 \times \underline{\quad} \\ & = \underline{\quad} \end{aligned}$$

$$\begin{aligned} \mathbf{b)} \quad & 10 \times (3^2 + 5^0) = 10 \times \underline{\hspace{2cm}} \\ & = 10 \times \underline{\quad} \\ & = \underline{\quad} \end{aligned}$$

$$\begin{aligned} \mathbf{c)} \quad & (-2)^3 + (-3)(4) = \underline{\quad} + \underline{\quad} \\ & = \underline{\quad} \end{aligned}$$

$$\begin{aligned} \mathbf{d)} \quad & (-3) + 4^0 \times (-3) = (-3) + \underline{\quad} \times (-3) \\ & = (-3) + \underline{\quad} \\ & = \underline{\quad} \end{aligned}$$

2.4 8. Write as a power.

a) $6^3 \times 6^7 = 6(\underline{\quad} + \underline{\quad})$
 $= 6\underline{\quad}$

b) $(-4)^2 \times (-4)^3 = (-4)\underline{\quad}$
 $= (-4)\underline{\quad}$

c) $(-2)^5 \times (-2)^4 = (-2)\underline{\quad}$
 $= (-2)\underline{\quad}$

d) $10^7 \times 10 = \underline{\quad}$
 $= \underline{\quad}$

9. Write as a power.

a) $5^7 \div 5^3 = 5(\underline{\quad} - \underline{\quad})$
 $= 5\underline{\quad}$

b) $\frac{10^5}{10^3} = \underline{\quad}$
 $= \underline{\quad}$

c) $(-6)^8 \div (-6)^2 = \underline{\quad}$
 $= \underline{\quad}$

d) $\frac{5^{10}}{5^6} = \underline{\quad}$
 $= \underline{\quad}$

e) $8^3 \div 8 = \underline{\quad}$
 $= \underline{\quad}$

f) $\frac{(-3)^4}{(-3)^0} = \underline{\quad}$
 $= \underline{\quad}$

2.5 10. Write as a power.

a) $(5^3)^4 = 5\underline{\quad} \times \underline{\quad}$
 $= 5\underline{\quad}$

b) $[(-3)^2]^6 = (-3)\underline{\quad} \times \underline{\quad}$
 $= (-3)\underline{\quad}$

c) $(8^2)^4 = \underline{\quad}$
 $= \underline{\quad}$

d) $[(-5)^5]^4 = \underline{\quad}$
 $= \underline{\quad}$

11. Write as a product or quotient of powers.

a) $(3 \times 5)^2 = 3\underline{\quad} \times 5\underline{\quad}$

b) $(2 \times 10)^5 = \underline{\quad}$

c) $[(-4) \times (-5)]^3 = \underline{\quad}$

d) $\left(\frac{4}{3}\right)^5 = \underline{\quad}$

e) $(12 \div 10)^4 = 12\underline{\quad} \div 10\underline{\quad}$

f) $[(-7) \div (-9)]^6 = \underline{\quad}$