UNIT 3

Rational Numbers

What You'll Learn

How to

- Identify positive and negative decimals and fractions as rational numbers
- Compare and order rational numbers
- Add, subtract, multiply, and divide rational numbers
- Solve problems that involve rational numbers
- Apply the order of operations with rational numbers

Why It's Important

Rational numbers are used by

- building contractors to measure and to estimate costs
- chefs to measure ingredients, plan menus, and estimate costs
- investment professionals to show changes in stock prices

Key Words

fraction
equivalent fraction
numerator
denominator
common denominator
multiple
common multiple
integer
decimal
repeating decimal
terminating decimal
rational number
reciprocal

TEACHER NOTE

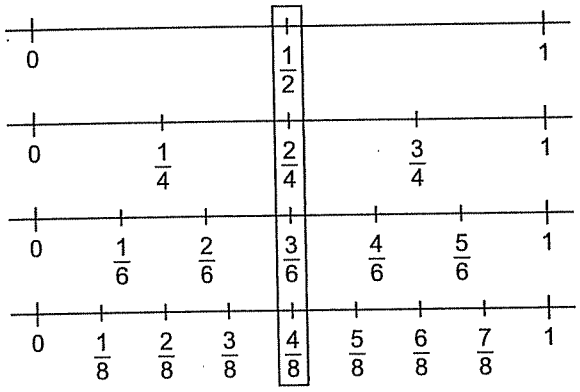
Some students just don't "get" fractions. Consider allowing students to use calculators. Or, suggest they convert fractions to decimals to simplify their work.

3.1 Skill Builder

Equivalent Fractions

$\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, and $\frac{4}{8}$ are **equivalent fractions.**

They represent the same distance on a number line.



TEACHER NOTE

Not all students need to review all 4 topics on pages 94 to 97. Assign work selectively.

Here is one way to find equivalent fractions. Multiply or divide the numerator and denominator by the same number.

$\frac{1}{2} \xrightarrow{\times 3} \frac{3}{6}$ $\quad \frac{4}{8} \xrightarrow{\div 2} \frac{2}{4}$

Multiplying or dividing both the numerator and denominator by the same number is like multiplying or dividing by 1. The original quantity is unchanged.

Check

1. Write 2 equivalent fractions. **Sample answers**

a) $\frac{7}{10} = \frac{14}{20} = \frac{28}{40}$ (×2, ×4)

b) $\frac{12}{15} = \frac{24}{30} = \frac{4}{5}$ (×2, ÷3)

2. Write an equivalent fraction with the given denominator.

a) $\frac{3}{5} = \frac{12}{20}$ (×4)

$5 \times 4 = 20$, so multiply the numerator and denominator by 4.

b) $\frac{1}{4} = \frac{3}{12}$ (×3)

$4 \times$ **3** $= 12$, so **multiply** the numerator and denominator by **3**.

c) $\frac{10}{15} = \frac{2}{3}$ (÷5)

$15 \div$ **5** $= 3$, so divide the numerator and denominator by **5**.

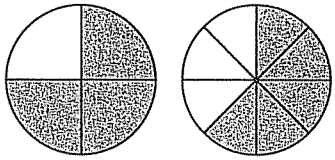
d) $\frac{20}{24} = \frac{5}{6}$ (÷4)

$24 \div$ **4** $= 6$, so **divide** the numerator and denominator by **4**.

Comparing Fractions

Here are 3 ways to compare $\frac{3}{4}$ and $\frac{5}{8}$.

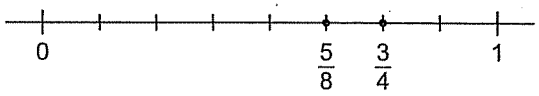
- Using area models:



$\frac{3}{4}$ $\frac{5}{8}$

Compare the shaded areas: $\frac{3}{4} > \frac{5}{8}$

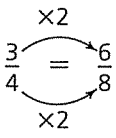
- Using number lines:



Numbers increase from left to right on a number line.

From the number line: $\frac{5}{8} < \frac{3}{4}$

- Writing equivalent fractions:

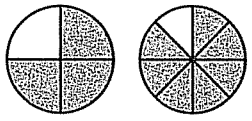


$\frac{5}{8} < \frac{6}{8}$; so, $\frac{5}{8} < \frac{3}{4}$

Check

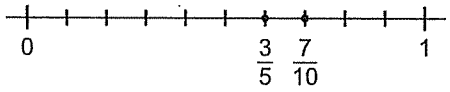
Compare the fractions in each pair. Write >, <, or =.

1. a) $\frac{7}{8} \geq \frac{3}{4}$

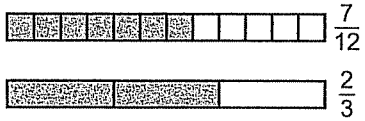


$\frac{3}{4}$ $\frac{7}{8}$

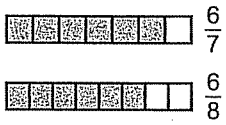
b) $\frac{3}{5} \leq \frac{7}{10}$



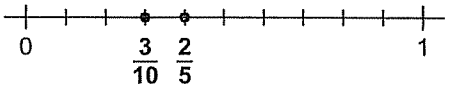
c) $\frac{7}{12} \leq \frac{2}{3}$



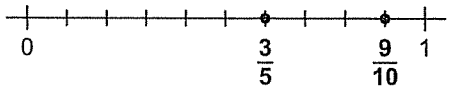
d) $\frac{6}{7} \geq \frac{6}{8}$



2. a) $\frac{2}{5} \geq \frac{3}{10}$



b) $\frac{3}{5} \leq \frac{9}{10}$



Common Denominators

To find a common denominator of $\frac{1}{2}$ and $\frac{2}{3}$:

Look for equivalent fractions with the same denominator.

List the multiples of 2: 2, 4, **6**, 8, 10, 12, 14, ...

List the multiples of 3: 3, **6**, 9, 12, 15, ...

6 is the least common multiple of 2 and 3. It is the simplest common denominator to work with.

Rewrite $\frac{1}{2}$ and $\frac{2}{3}$ with denominator 6.

$\frac{1}{2} \overset{\times 3}{=} \frac{3}{6}$ $\quad$ $\frac{2}{3} \overset{\times 2}{=} \frac{4}{6}$

Equivalent fractions help us compare, add, or subtract fractions.

Check

1. Write equivalent fraction pairs with a common denominator.

a) $\frac{1}{2}$ and $\frac{3}{8}$

Multiples of 2: 2, 4, 6, 8, 10, ...

Multiples of 8: 8, 16, ...

A common denominator is **8**.

So, $\frac{1}{2} \overset{\times 4}{=} \frac{4}{8}$ and $\frac{3}{8} \overset{\times 1}{=} \frac{3}{8}$

b) $\frac{3}{4}$ and $\frac{5}{6}$

Multiples of 4: **4, 8, 12, ...**

Multiples of 6: **6, 12, ...**

A common denominator is 12.

So, $\frac{3}{4} \overset{\times 3}{=} \frac{9}{12}$ and $\frac{5}{6} \overset{\times 2}{=} \frac{10}{12}$

c) $\frac{3}{5}$ and $\frac{2}{3}$

Multiples of **5**: **5, 10, 15, ...**

Multiples of **3**: **3, 6, 9, 12, 15, ...**

A common denominator is 15.

So, $\frac{3}{5} \overset{\times 3}{=} \frac{9}{15}$ and $\frac{2}{3} \overset{\times 5}{=} \frac{10}{15}$

2. Compare each pair of fractions from question 1.

a) $\frac{1}{2}$ and $\frac{3}{8}$. Since $\frac{4}{8} > \frac{3}{8}$, $\frac{1}{2} \geq \frac{3}{8}$

b) $\frac{3}{4}$ and $\frac{5}{6}$. Since $\frac{9}{12} < \frac{10}{12}$, $\frac{3}{4} \leq \frac{5}{6}$

c) $\frac{3}{5}$ and $\frac{2}{3}$. Since $\frac{9}{15} < \frac{10}{15}$, $\frac{3}{5} \leq \frac{2}{3}$

TEACHER NOTE

For related review, use Master 3.25a from the *Math Makes Sense 9* ProGuide™.

Converting between Fractions and Decimals

- Fractions to decimals

  The fraction bar represents division. For example:

  $\frac{1}{6}$ means $1 \div 6$

  Use a calculator:

  $1 \div 6 = 0.166\,666\ldots$

  $= 0.1\overline{6}$

  So, $\frac{1}{6} = 0.1\overline{6}$

  The bar over the 6 means that 6 repeats.

  $0.1\overline{6}$ is a **repeating decimal.**

  $\frac{7}{8}$ means $7 \div 8$

  Use a calculator:

  $7 \div 8 = 0.875$

  So, $\frac{7}{8} = 0.875$

  0.875 is a **terminating decimal.**

- Decimals to fractions

  Use place value. For example:

  0.7 means 7 tenths.

  So, $0.7 = \frac{7}{10}$

  0.23 means 23 hundredths

  So, $0.23 = \frac{23}{100}$

Check

1. Write each fraction as a decimal.

a) $\frac{3}{4} = 3 \div 4$

$= \mathbf{0.75}$

b) $\frac{2}{3} = \mathbf{2 \div 3}$

$= \mathbf{0.\overline{6}}$

c) $\frac{5}{8} = \mathbf{5 \div 8}$

$= \mathbf{0.625}$

d) $\frac{5}{9} = 5 \div 9$

$= \mathbf{0.\overline{5}}$

e) $4\frac{1}{5} = 4 + \frac{1}{5}$

$= 4 + \mathbf{1 \div 5}$

$= 4 + \mathbf{0.2}$

$= \mathbf{4.2}$

f) $2\frac{1}{3} = 2 + \mathbf{\frac{1}{3}}$

$= 2 + \mathbf{1 \div 3}$

$= 2 + \mathbf{0.\overline{3}}$

$= \mathbf{2.\overline{3}}$

2. Which numbers in question 1 are:

a) repeating decimals? $\mathbf{\frac{2}{3}, \frac{5}{9}, 2\frac{1}{3}}$

b) terminating decimals? $\mathbf{\frac{3}{4}, \frac{5}{8}, 4\frac{1}{5}}$

3. Write each decimal as a fraction.

a) $0.3 = \mathbf{\frac{3}{10}}$

b) $0.9 = \mathbf{\frac{9}{10}}$

c) $0.11 = \mathbf{\frac{11}{100}}$

d) $0.87 = \mathbf{\frac{87}{100}}$

e) $1.5 = \mathbf{1\frac{5}{10}}$, **or** $\mathbf{1\frac{1}{2}}$

f) $5.7 = \mathbf{5\frac{7}{10}}$

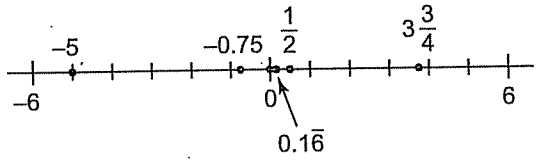
3.1 What Is a Rational Number?

FOCUS **Compare and order rational numbers.**

Rational numbers include:

- integers
- positive and negative fractions
- positive and negative mixed numbers
- repeating and terminating decimals

Here is a number line that displays some rational numbers.



Example 1 Finding a Rational Number between Two Given Numbers

Find 2 rational numbers between $2\frac{1}{3}$ and $3\frac{3}{4}$.

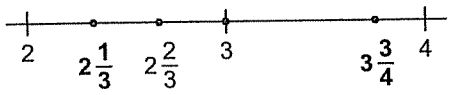
Solution

Label a number line from 2 to 4.

$2\frac{1}{3}$ is one-third of the way from 2 to 3.

$3\frac{3}{4}$ is three-quarters of the way from 3 to 4.

From the number line, 2 rational numbers between $2\frac{1}{3}$ and $3\frac{3}{4}$ are: $2\frac{2}{3}$ and 3



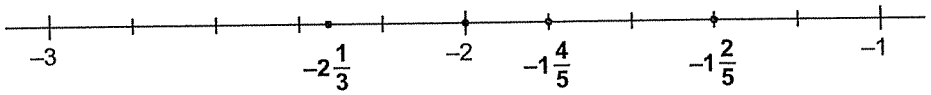
There are many correct solutions. Which ones can you name?

Check

1. Find 2 rational numbers between each pair of numbers. **Sample answers**

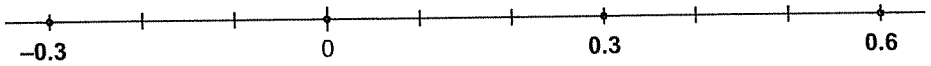
Plot points to show $-1\frac{2}{5}$ and $-2\frac{1}{3}$.

a) $-2\frac{1}{3}$ and $-1\frac{2}{5}$



From the number line, 2 values between $-2\frac{1}{3}$ and $-1\frac{2}{5}$ are: $-1\frac{4}{5}$ and -2

b) −0.3 and 0.6



From the number line, 2 values between −0.3 and 0.6 are: 0 and 0.3

Example 2 Comparing Rational Numbers on a Number Line

Order each set of rational numbers from least to greatest.

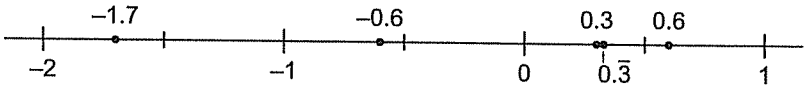
a) $0.3, 0.\overline{3}, -1.7, 0.6, -0.6$

b) $3\frac{1}{4}, -\frac{3}{4}, -\frac{4}{8}, 1\frac{3}{4}, -2\frac{3}{8}$

TEACHER NOTE
If you have suggested students use a calculator to convert fractions to decimals, let them know whether you will accept their answers in decimal form.

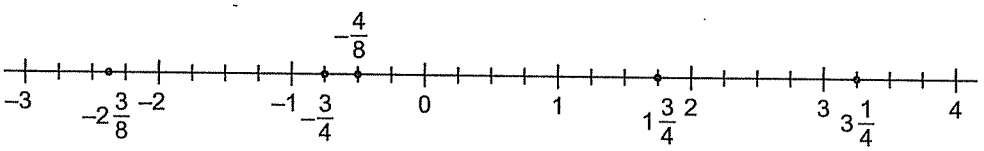
Solution

a) Plot the numbers on a number line.
To plot 0.3 and $0.\overline{3}$, think: $0.\overline{3} = 0.3333...$
So, $0.\overline{3}$ is slightly greater than 0.3.



From the number line, the order from least to greatest is: $-1.7, -0.6, 0.3, 0.\overline{3}, 0.6$

b) Plot the numbers on a number line.

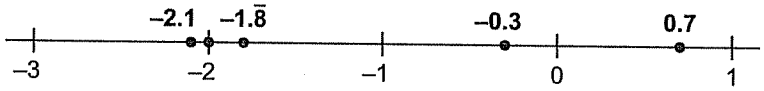


From the number line, the order from least to greatest is: $-2\frac{3}{8}, -\frac{3}{4}, -\frac{4}{8}, 1\frac{3}{4}, 3\frac{1}{4}$

Check

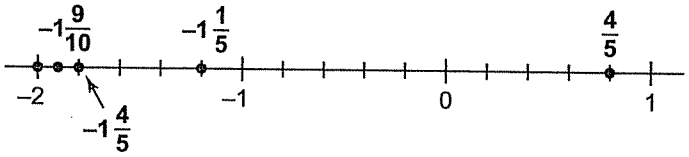
1. Order each set of numbers from least to greatest.

a) $-1.\overline{8}, 0.7, -2, -2.1, -0.3$



From the number line, the order from least to greatest is: **$-2.1, -2, -1.\overline{8}, -0.3, 0.7$**

b) $-1\frac{9}{10}, -2, -1\frac{4}{5}, \frac{4}{5}, -1\frac{1}{5}$



The number line is divided in fifths to help you plot the numbers.

From the number line, the order from least to greatest is: **$-2, -1\frac{9}{10}, -1\frac{4}{5}, -1\frac{1}{5}, \frac{4}{5}$**

Practice

1. Write each rational number as a decimal.

TEACHER NOTE
Question 1 highlights an important point about interpreting fractions involving signs.

a) $\frac{3}{5} = \frac{3}{5} \div \frac{5}{}$
$= 0.6$

b) $\frac{5}{3} = \frac{5 \div 3}{}$
$= 1.\overline{6}$

c) $-\frac{3}{5} = -(\frac{3}{} \div \frac{5}{})$
$= -0.6$

d) $\frac{-3}{5} = (\frac{-3}{}) \div \frac{5}{}$
$= -0.6$

e) $\frac{-5}{3} = (\frac{-5}{}) \div \frac{3}{}$
$= -1.\overline{6}$

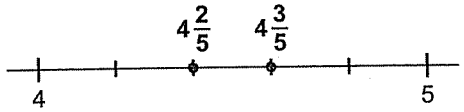
f) $\frac{3}{-5} = \frac{3 \div (-5)}{}$
$= -0.6$

Look for matching answers. What conclusion can you make?

Parts c, d, and f match. This shows that $-\frac{3}{5}, \frac{-3}{5}, \frac{3}{-5}$ are all equal.

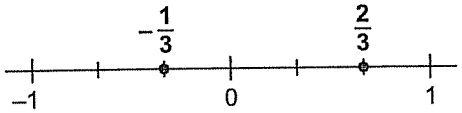
2. Plot and compare each pair of rational numbers.

a) $4\frac{2}{5}$ and $4\frac{3}{5}$



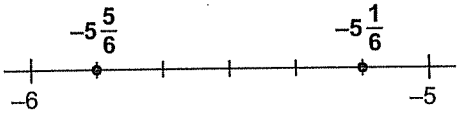
From the number line, $4\frac{2}{5} \leq 4\frac{3}{5}$

b) $\frac{2}{3}$ and $-\frac{1}{3}$



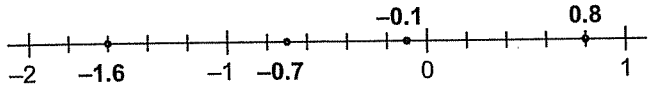
From the number line, $\frac{2}{3} > -\frac{1}{3}$

c) $-5\frac{5}{6}$ and $-5\frac{1}{6}$



From the number line, $-5\frac{5}{6} < -5\frac{1}{6}$

3. a) Write a decimal to match each point on the number line.

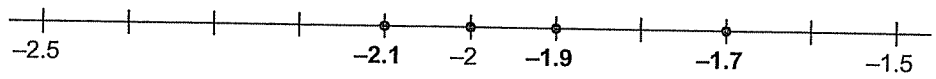


b) Write the numbers in part a from least to greatest.

$-1.6, -0.7, -0.1, 0.8$

4. Find 2 rational numbers between each pair of numbers. **Sample answers**

a) -2.1 and -1.7



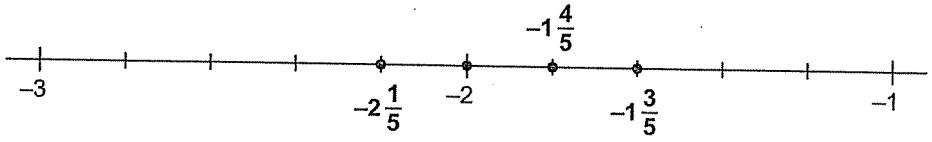
Two possible numbers are: **-2 and -1.9**

b) 4.1 and 4.4

Start by plotting the given values on the number line.

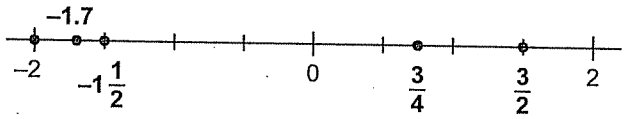
Two possible numbers are: **4.2 and 4.3**

c) $-1\frac{3}{5}$ and $-2\frac{1}{5}$



Two possible numbers are: -2 and $-1\frac{4}{5}$

5. Order these rational numbers from least to greatest.
$-1\frac{1}{2}, \frac{3}{2}, -1.7, -2, \frac{3}{4}$



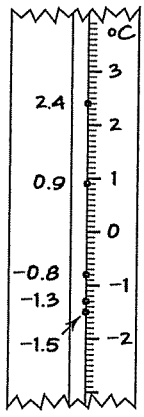
Estimate to place numbers where necessary.

From least to greatest: **$-2, -1.7, -1\frac{1}{2}, \frac{3}{4}, \frac{3}{2}$**

6. Kiki recorded the temperatures at the same time each day over a 5-day period.

$-0.8°C, -1.3°C, 2.4°C, -1.5°C, 0.9°C$

Order the temperatures from lowest to highest:
$-1.5°C, -1.3°C, -0.8°C, 0.9°C, 2.4°C$



TEACHER NOTE

Next Steps: Direct students to questions 8, 9, 12, and 16 on pages 101 and 102 of the Student Text.

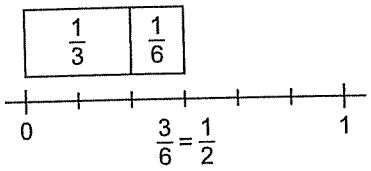
For students experiencing success, assign *Practice* question 13.

3.2 Skill Builder

Adding Fractions

Here are 2 ways to add $\frac{1}{3}$ and $\frac{1}{6}$.

- Using fraction strips on a number line:
  Place the fraction strips end to end, starting at 0.



From the number line: $\frac{1}{3} + \frac{1}{6} = \frac{3}{6}$, or $\frac{1}{2}$

- Using common denominators:
  $\frac{1}{3}$ is the same as $\frac{2}{6}$.

  So, $\frac{1}{3} + \frac{1}{6} = \frac{2}{6} + \frac{1}{6}$

  $= \frac{3}{6}$, or $\frac{1}{2}$

Some additions give answers that are greater than 1.

$\frac{2}{3} + \frac{1}{2} = \frac{4}{6} + \frac{3}{6}$

$= \frac{7}{6}$ ← improper fraction

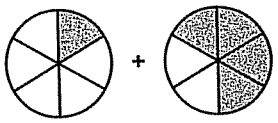
$= 1\frac{1}{6}$ ← mixed number

Rewrite the improper fraction as a mixed number: divide 6 into 7 to see that there is 1 whole, and 1 sixth left over.

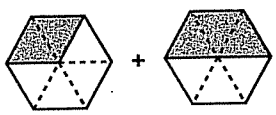
Check

1. Find each sum. Use diagrams to show your thinking.

a) $\frac{1}{6} + \frac{4}{6} = \underline{\mathbf{\frac{5}{6}}}$



b) $\frac{1}{3} + \frac{1}{2} = \underline{\mathbf{\frac{5}{6}}}$



2. Find each sum. Use the method you like best.

a) $\frac{2}{5} + \frac{4}{5} = \underline{\mathbf{\frac{6}{5}}}$, or $\underline{\mathbf{1\frac{1}{5}}}$

b) $\frac{2}{4} + \frac{5}{8} = \underline{\mathbf{\frac{4}{8} + \frac{5}{8}}}$

$= \underline{\mathbf{\frac{9}{8}}}$, or $\underline{\mathbf{1\frac{1}{8}}}$

Adding Mixed Numbers

Mixed numbers combine whole numbers and fractions.

We can add numbers in any order without changing the answer.

Add: $1\frac{1}{8} + 3\frac{3}{4}$

Add the whole numbers and add the fractions.

$1\frac{1}{8} + 3\frac{3}{4} = 1 + 3 + \frac{1}{8} + \frac{3}{4}$ A common denominator is 8.

$= 1 + 3 + \frac{1}{8} + \frac{6}{8}$

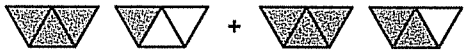
$= 4 + \frac{7}{8}$

$= 4\frac{7}{8}$

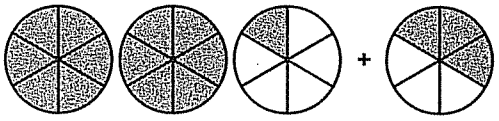
Check

1. Find each sum. Use diagrams to show your thinking.

a) $1\frac{1}{3} + 1\frac{2}{3} =$ **3**



b) $2\frac{1}{6} + \frac{1}{2} =$ **$2\frac{4}{6}$, or $2\frac{2}{3}$**



2. Find each sum.
Use the method you like best.

a) $3\frac{2}{7} + 2\frac{3}{7} = 3 + 2 + \frac{2}{7} + \frac{3}{7}$

$= 5 + \frac{5}{7}$

$= 5\frac{5}{7}$

b) $4\frac{1}{9} + 1\frac{2}{3} = 4 + 1 + \frac{1}{9} + \frac{2}{3}$

$= 5 + \frac{1}{9} + \frac{6}{9}$

$= 5 + \frac{7}{9}$

$= 5\frac{7}{9}$

3.2 Adding Rational Numbers

FOCUS **Solve problems by adding rational numbers.**

Integers and fractions are rational numbers.
So, you can use strategies for adding integers, and strategies for adding fractions, to add rational numbers.

Example 1 Adding Rational Numbers on a Number Line

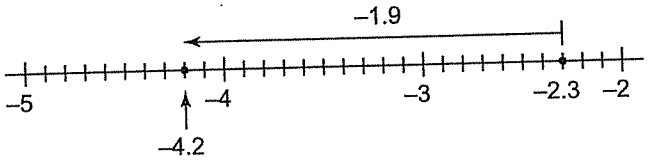
a) $-2.3 + (-1.9)$ **b)** $-\frac{1}{2} + \left(-\frac{5}{4}\right)$

Solution

a) $-2.3 + (-1.9)$

Use a number line divided in tenths.

Start at -2.3. To add -1.9, move 1.9 to the left.



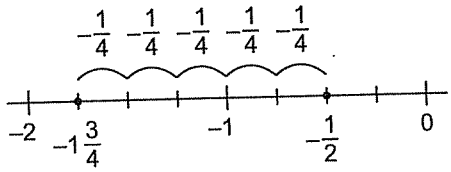
When we add a negative number, we move to the left. When we add a positive number, we move to the right.

So, $-2.3 + (-1.9) = -4.2$.

b) $-\frac{1}{2} + \left(-\frac{5}{4}\right)$

Use a number line divided into fourths.

Start at $-\frac{1}{2}$. To add $-\frac{5}{4}$, move $\frac{5}{4}$ to the left.

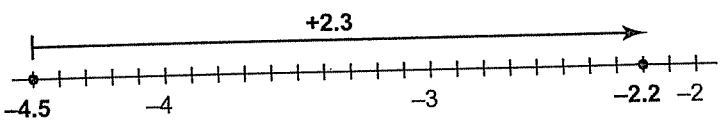


So, $-\frac{1}{2} + \left(-\frac{5}{4}\right) = -1\frac{3}{4}$.

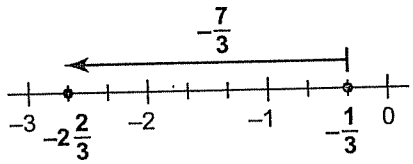
Check

1. Use a number line to add.

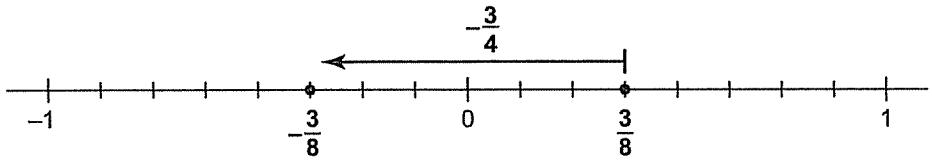
a) $-4.5 + 2.3 =$ **−2.2**



b) $-\frac{1}{3} + \left(-\frac{7}{3}\right) =$ **$-2\frac{2}{3}$**



c) $\frac{3}{8} + \left(-\frac{3}{4}\right) =$ $-\frac{3}{8}$



Example 2 Adding Fractions without a Number Line

Add: $-\frac{2}{5} + \left(-\frac{1}{2}\right)$

Solution

To find $-\frac{2}{5} + \left(-\frac{1}{2}\right)$, look for a common denominator.

Use a common denominator of 10.

Multiples of 5: 5, 10, 15, ...
Multiples of 2: 2, 4, 6, 8, 10, ...

$-\frac{2}{5} \overset{\times 2}{=} -\frac{4}{10}$ and $-\frac{1}{2} \overset{\times 5}{=} -\frac{5}{10}$

So, $-\frac{2}{5} + \left(-\frac{1}{2}\right) = -\frac{4}{10} + \left(-\frac{5}{10}\right)$

$= -\frac{9}{10}$

Think of integer addition: $(-4) + (-5) = -9$

Check

1. Add.

a) $-\frac{7}{12} + \frac{1}{6}$ Use a common denominator of **12**. $\frac{1}{6} \overset{\times 2}{=} \frac{2}{12}$

$= -\frac{7}{12} + \frac{2}{12}$

$= -\frac{5}{12}$

b) $\frac{3}{5} + \left(-\frac{2}{3}\right)$ Use a common denominator of **15**. $\frac{3}{5} \overset{\times 3}{=} \frac{9}{15}$ and $-\frac{2}{3} \overset{\times 5}{=} -\frac{10}{15}$

$= \frac{9}{15} + \left(-\frac{10}{15}\right)$

$= -\frac{1}{15}$

Example 3 Adding Mixed Numbers

Calculate: $-2\frac{1}{8} + 3\frac{1}{3}$

Solution

Estimate first to predict the answer:

$-2\frac{1}{8} + 3\frac{1}{3}$ is about $-2 + 3$, or 1.

We expect an answer close to 1.

To calculate, add the whole numbers and add the fractions.
Keep the signs with each part of the mixed number.

$-2\frac{1}{8} + 3\frac{1}{3} = (-2) + 3 + \left(-\frac{1}{8}\right) + \frac{1}{3}$ Use a common denominator of 24.

$-\frac{1}{8} \xrightarrow{\times 3} -\frac{3}{24}$ and $\frac{1}{3} \xrightarrow{\times 8} \frac{8}{24}$

So, $-2\frac{1}{8} + 3\frac{1}{3} = (-2) + 3 + \left(-\frac{3}{24}\right) + \frac{8}{24}$

$= 1 + \frac{5}{24}$

$= 1\frac{5}{24}$

Check: the answer is reasonably close to the original estimate of 1.

Check

1. Find each sum.

a) $-1\frac{5}{16} + 3\frac{3}{8} = \underline{(-1)} + \underline{3} + \underline{\left(-\frac{5}{16}\right)} + \underline{\frac{3}{8}}$

Use a common denominator of **16**.

$= \underline{(-1)} + \underline{3} + \underline{\left(-\frac{5}{16}\right)} + \underline{\frac{6}{16}}$

$\frac{3}{8} \xrightarrow{\times 2} \frac{6}{16}$

$= \underline{2} + \underline{\frac{1}{16}}$

$= \underline{2\frac{1}{16}}$

Estimate to check if your answer is reasonable.

b) $2\frac{3}{5} + 1\frac{1}{4} = \underline{2} + \underline{1} + \underline{\frac{3}{5}} + \underline{\frac{1}{4}}$

$= \underline{2} + \underline{1} + \underline{\frac{12}{20}} + \underline{\frac{5}{20}}$

$= \underline{3} + \underline{\frac{17}{20}}$

$= \underline{3\frac{17}{20}}$

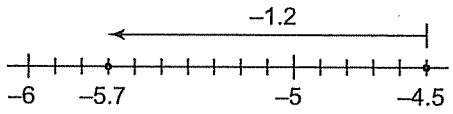
Use a common denominator of **20**.

$\frac{3}{5} \overset{\times 4}{=} \frac{12}{20}$ and $\frac{1}{4} \overset{\times 5}{=} \frac{5}{20}$

Practice

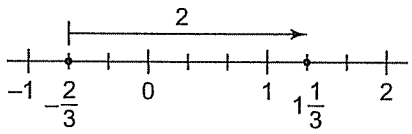
1. Write the addition statement shown by each number line.

a)



−4.5 + (**−1.2**) = **−5.7**

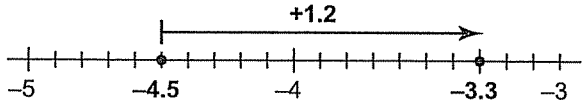
b)



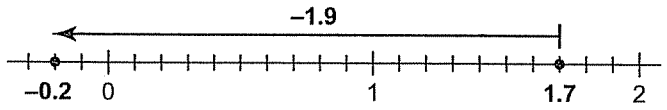
$\underline{-\frac{2}{3} + 2 = 1\frac{1}{3}}$

2. Use the number line to add.

a) −4.5 + (1.2) = **−3.3**



b) 1.7 + (−1.9) = **−0.2**



3. Add.

a) **i)** 4 + 6 = **10** **ii)** 4.1 + 6.4 = **10.5** **iii)** $\frac{4}{11} + \frac{6}{11} = \frac{10}{11}$

b) **i)** 4 + (−6) = **−2** **ii)** 4.1 + (−6.4) = **−2.3** **iii)** $\frac{4}{11} + \left(-\frac{6}{11}\right) = -\frac{2}{11}$

c) **i)** −4 + 6 = **2** **ii)** −4.1 + 6.4 = **2.3** **iii)** $-\frac{4}{11} + \frac{6}{11} = \frac{2}{11}$

d) **i)** −4 + (−6) = **−10** **ii)** −4.1 + (−6.4) = **−10.5** **iii)** $-\frac{4}{11} + \left(-\frac{6}{11}\right) = -\frac{10}{11}$

4. Find each sum.

a) $-4.6 + 5.8 =$ **1.2**

b) $2.3 + (-4.6) =$ **-2.3**

c) $-0.3 + (-6.2) =$ **-6.5**

d) $(-26.5) + (-18.1) =$ **-44.6**

5. Find each sum.

a) $-\frac{1}{3} + \frac{5}{9}$

$= -\frac{3}{9} + \frac{5}{9}$

$= \frac{2}{9}$

b) $\frac{1}{3} + \left(-\frac{2}{5}\right)$

$= \frac{5}{15} + \left(-\frac{6}{15}\right)$

$= -\frac{1}{15}$

c) $-\frac{3}{8} + \left(-\frac{1}{3}\right)$

$= -\frac{9}{24} + \left(-\frac{8}{24}\right)$

$= -\frac{17}{24}$

6. Find each sum.

Look for a common denominator first.

a) $-2\frac{2}{5} + 6\frac{1}{2}$

$= (-2 + 6) + \left(-\frac{2}{5}\right) + \frac{1}{2}$

$= (-2 + 6) + \left(-\frac{4}{10} + \frac{5}{10}\right)$

$= 4 + \frac{1}{10}$

$= 4\frac{1}{10}$

b) $-1\frac{1}{6} + \left(-3\frac{1}{4}\right)$

$= (-1) + (-3) + \left(-\frac{1}{6}\right) + \left(-\frac{1}{4}\right)$

$= (-1) + (-3) + \left(-\frac{2}{12}\right) + \left(-\frac{3}{12}\right)$

$= (-4) + \left(-\frac{5}{12}\right)$

$= -4\frac{5}{12}$

c) $\left(-3\frac{1}{3}\right) + \left(-5\frac{1}{7}\right)$

$= (-3) + (-5) + \left(-\frac{1}{3}\right) + \left(-\frac{1}{7}\right)$

$= (-3) + (-5) + \left(-\frac{7}{21}\right) + \left(-\frac{3}{21}\right)$

$= (-8) + \left(-\frac{10}{21}\right)$

$= -8\frac{10}{21}$

TEACHER NOTE

Next Steps: Direct students to *Practice* questions 6, 7, and 9 on pages 111 to 112 of the Student Text.

For students experiencing success, assign *Practice* questions 15, 17, 18.

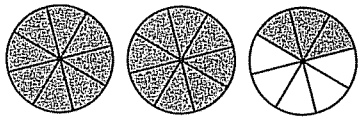
3.3 Skill Builder

Converting Mixed Numbers to Improper Fractions

Here are 2 ways to write $2\frac{3}{8}$ as an improper fraction.

- Make a diagram to show $2\frac{3}{8}$.

  Count individual parts.



8... 16... 19

Think of the diagram above:

2 × 8 + 3

↑ 2 whole circles shaded

↑ 8 pieces in each circle

↑ plus another 3 pieces

- Use calculation.

$$2\frac{3}{8} = \frac{2 \times 8 + 3}{8}$$

$$= \frac{19}{8}$$

$$2\frac{3}{8} = 2 + \frac{3}{8}$$

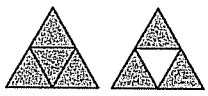
$$= \frac{16}{8} + \frac{3}{8}$$

$$= \frac{19}{8}$$

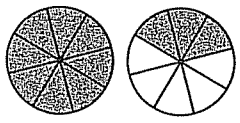
Check

1. Write a mixed number and an improper fraction to show each shaded quantity.

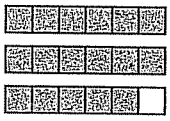
a) $1\frac{3}{4}$ or $\frac{7}{4}$



b) $1\frac{3}{8}$ or $\frac{11}{8}$



c) $2\frac{5}{6}$ or $\frac{17}{6}$



d) $3\frac{4}{9}$ or $\frac{31}{9}$

2. Write each mixed number as an improper fraction.

a) $1\frac{2}{5} = \mathbf{1} + \frac{2}{5}$

$= \frac{\mathbf{5}}{5} + \frac{2}{5}$

$= \frac{\mathbf{7}}{5}$

b) $2\frac{2}{3} = \mathbf{2} + \frac{\mathbf{2}}{\mathbf{3}}$

$= \frac{\mathbf{6}}{3} + \frac{\mathbf{2}}{\mathbf{3}}$

$= \frac{\mathbf{8}}{\mathbf{3}}$

c) $5\frac{3}{4} = \mathbf{5} + \frac{\mathbf{3}}{\mathbf{4}}$

$= \frac{\mathbf{20}}{4} + \frac{\mathbf{3}}{\mathbf{4}}$

$= \frac{\mathbf{23}}{\mathbf{4}}$

3.3 Subtracting Rational Numbers

FOCUS **Solve problems by subtracting rational numbers.**

To subtract an integer, we add its opposite.

- $-5 - 2$ is the same as $-5 + (-2)$.

  So, $-5 - 2 = -5 + (-2)$
  $= -7$

- $-5 - (-2)$ is the same as $-5 + (+2)$

  So, $-5 - (-2) = -5 + (+2)$
  $= -3$

We can use the same strategy to subtract rational numbers.

Subtracting Rational Numbers
To subtract a rational number, add its opposite.

Example 1 Subtracting Rational Numbers in Fraction Form

Subtract: $\frac{1}{3} - \frac{5}{6}$

Solution

$\frac{1}{3} - \frac{5}{6}$	Add the opposite.
$= \frac{1}{3} + \left(-\frac{5}{6}\right)$	Use 6 as a common denominator.
$= \frac{2}{6} + \left(-\frac{5}{6}\right)$	Think of integer addition: $2 + (-5) = -3$
$= -\frac{3}{6}$	Write the answer in simplest form.
$= -\frac{1}{2}$	

Check

1. Subtract.

a) $-\frac{1}{2} - \frac{7}{8} = -\frac{1}{2} + \left(-\frac{7}{8}\right)$
$= -\frac{4}{8} + \left(-\frac{7}{8}\right)$
$= \frac{-11}{8}$
$= -1\frac{3}{8}$

b) $\frac{4}{5} - \left(-\frac{2}{3}\right) = \frac{4}{5} + \left(\frac{2}{3}\right)$
$= \frac{12}{15} + \left(\frac{10}{15}\right)$
$= \frac{22}{15}$
$= 1\frac{7}{15}$

TEACHER NOTE
For students who struggle with fractions, consider suggesting they convert to decimals to solve problems. Clarify your expectations if you want fraction form for the final answer.

Example 2 Subtracting Rational Numbers in Mixed Number Form

Subtract: $\frac{3}{4} - 2\frac{5}{8}$

Solution

$\frac{3}{4} - 2\frac{5}{8}$ Write $2\frac{5}{8}$ as an improper fraction.

$= \frac{3}{4} - \frac{21}{8}$ Use 8 as a common denominator.

$= \frac{6}{8} - \frac{21}{8}$ Add the opposite.

$= \frac{6}{8} + \left(-\frac{21}{8}\right)$

$= -\frac{15}{8}$, or $-1\frac{7}{8}$

Check

1. Find the difference.

a) $-\frac{13}{15} - 1\frac{1}{5}$ Write $1\frac{1}{5}$ as an improper fraction.

$= -\frac{13}{15} - \frac{\mathbf{6}}{\mathbf{5}}$ Use **15** as a common denominator.

$= -\frac{13}{15} - \frac{\mathbf{18}}{15}$ Add the opposite.

$= -\frac{13}{15} + \left(-\frac{\mathbf{18}}{15}\right)$

$= \mathbf{-\frac{31}{15}}$ Write the answer as a mixed number.

$= \mathbf{-2\frac{1}{15}}$

b) $-2\frac{3}{8} - 3\frac{1}{2}$ Rewrite $-2\frac{3}{8}$ and $3\frac{1}{2}$ as improper fractions.

$= \mathbf{-\frac{19}{8}} - \mathbf{\frac{7}{2}}$ Use **8** as a common denominator.

$= \mathbf{-\frac{19}{8}} - \mathbf{\frac{28}{8}}$ Add the opposite.

$= \mathbf{-\frac{19}{8}} + \mathbf{\left(-\frac{28}{8}\right)}$

$= \mathbf{-\frac{47}{8}}$ Write the answer as a mixed number.

$= \mathbf{-5\frac{7}{8}}$

Example 3 Solving a Problem by Subtracting Rational Numbers

In Alberta:

- The lowest temperature ever recorded was −61.1°C at Fort Vermilion in 1911.
- The highest temperature was 43.3°C at Bassano Dams in 1931.

What is the difference between these temperatures?

Solution

Subtract to find the difference between the temperatures.

$43.3 - (-61.1)$ Add the opposite.

$= 43.3 + (61.1)$

$= 104.4$

The difference between the temperatures is 104.4°C.

Use mental math to check.
40 + 60 = 100
3.3 + 1.1 = 4.4
100 + 4.4 = 104.4

Check

1. The lowest temperature ever recorded on Earth was −89.2°C in Antarctica.
The highest temperature ever recorded is 57.8°C in Libya.
What is the difference between these temperatures?

57.8 − (**−89.2**) = **57.8** + (**+89.2**)

= **147**

The difference between the temperatures is **147** °C.

Practice

1. Subtract.

a) 1.6 − 3.9 = **−2.3**

b) 1.6 − (−3.9) = **5.5**

c) −2.4 − 4.5 = **−6.9**

d) 2.4 − (−4.5) = **6.9**

2. Draw lines to join matching subtraction sentences, addition sentences, and answers.

Subtraction sentence	Addition sentence	Answer
2.7 − 9.7	2.7 + 9.7	−12.4
−2.7 − 9.7	2.7 + (−9.7)	−7
−2.7 − (−9.7)	−2.7 + (−9.7)	7
2.7 − (−9.7)	−2.7 + 9.7	12.4

3. Find each difference.

a) $7.1 - 4.7 =$ **2.4**

b) $-3.2 - 1.9 =$ **−5.1**

c) $26.2 - (-8.4) =$ **34.6**

d) $(-8.6) - (-7.2) =$ **−1.4**

Estimate to check if your answers are reasonable.

4. Subtract.

a) i) $6 - 3 =$ **3**

ii) $6.3 - 3.1 =$ **3.2**

iii) $\frac{6}{7} - \frac{3}{7} =$ **$\frac{3}{7}$**

b) i) $-6 - 3 =$ **−9**

ii) $-6.3 - 3.1 =$ **−9.4**

iii) $-\frac{6}{7} - \frac{3}{7} =$ **$-\frac{9}{7}$, or $-1\frac{2}{7}$**

c) i) $6 - (-3) =$ **9**

ii) $6.3 - (-3.1) =$ **9.4**

iii) $\frac{6}{7} - \left(-\frac{3}{7}\right) =$ **$\frac{9}{7}$, or $1\frac{2}{7}$**

d) i) $-6 - (-3) =$ **−3**

ii) $-6.3 - (-3.1) =$ **−3.2**

iii) $-\frac{6}{7} - \left(-\frac{3}{7}\right) =$ **$-\frac{3}{7}$**

5. Determine each difference.

a) $\frac{3}{5} - \left(-\frac{1}{3}\right) = \frac{3}{5} + \frac{1}{3}$

$= \frac{9}{15} + \frac{5}{15}$

$= \frac{14}{15}$

b) $-\frac{17}{20} - \frac{3}{2} = -\frac{17}{20} + \left(-\frac{3}{2}\right)$

$= -\frac{17}{20} + \left(-\frac{30}{20}\right)$

$= -\frac{47}{20}$, or $-2\frac{7}{20}$

c) $\frac{9}{5} - \frac{7}{4} = \frac{9}{5} + \left(-\frac{7}{4}\right)$

$= \frac{36}{20} + \left(-\frac{35}{20}\right)$

$= \frac{1}{20}$

6. Calculate.

a) $2\frac{1}{6} - 1\frac{1}{3} = \frac{13}{6} - \frac{4}{3}$

$= \frac{13}{6} + \left(-\frac{4}{3}\right)$

$= \frac{13}{6} + \left(-\frac{8}{6}\right)$

$= \frac{5}{6}$

b) $1\frac{1}{2} - \left(-2\frac{1}{3}\right) = \frac{3}{2} - \left(-\frac{7}{3}\right)$

$= \frac{3}{2} + \frac{7}{3}$

$= \frac{9}{6} + \frac{14}{6}$

$= \frac{23}{6}$

TEACHER NOTE

Next Steps: Direct students to questions 6, 7, 8 and 11 on pages 119 to 120 of the Student Text.

7. Jenny has a gift card with $24.50 left on it. She makes purchases totaling $42.35. What amount does Jenny still owe the cashier after using the gift card?

Subtraction sentence: **42.35** − **24.50** = **17.85**

Jenny still owes the cashier $**17.85**.

CHECKPOINT

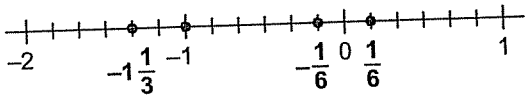
Can you ...

- Compare and order rational numbers?
- Add and subtract rational numbers?
- Solve problems by adding and subtracting rational numbers?

3.1 **1.** Find 2 rational numbers between each pair of numbers. **Sample answers**

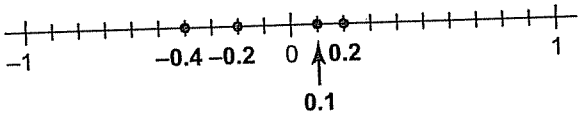
a) $-1\frac{1}{3}$ and $\frac{1}{6}$

Plot each number on the number line.



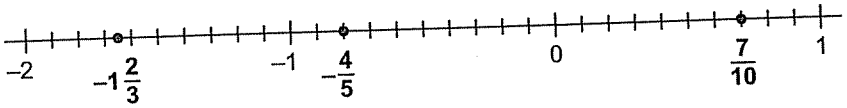
From the number line, 2 values between $-1\frac{1}{3}$ and $\frac{1}{6}$ are: $-\frac{1}{6}$ and **−1**

b) −0.4 and 0.2



From the number line, 2 values between −0.4 and 0.2 are: **−0.2** and **0.1**

2. Use the number line to order the fractions from least to greatest: $-1\frac{2}{3}$, $\frac{7}{10}$, $-\frac{4}{5}$



For least to greatest, read the points from **left** to **right**: $-1\frac{2}{3}$, $-\frac{4}{5}$, $\frac{7}{10}$

3. a) Write each number as a decimal.

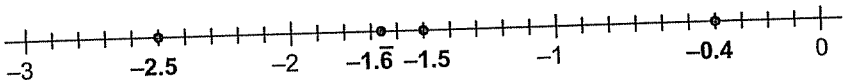
$-\frac{2}{5} =$ **−0.4** $\quad -1\frac{1}{2} =$ **−1.5**

$-\frac{5}{3} =$ **$-1.\overline{6}$** $\quad -\frac{5}{2} =$ **−2.5**

$-\frac{2}{5}$ means $-(2 \div 5)$.

b) Order the decimals in part a from least to greatest.

Use the number line to help you.



From least to greatest: **$-2.5, -1.\overline{6}, -1.5, -0.4$**

3.2 **4.** Find each sum.

a) $6.5 + (-4.2) =$ **2.3**

b) $-13.6 + (-7.9) =$ **−21.5**

5. Find each sum. Use equivalent fractions.

a) $-\frac{3}{8} + \frac{1}{4} = -\frac{3}{8} + \frac{2}{8}$

$= -\frac{1}{8}$

b) $\frac{3}{8} + \frac{1}{4} = \frac{3}{8} + \frac{2}{8}$

$= \frac{5}{8}$

c) $-\frac{3}{8} + \left(-\frac{1}{4}\right) = -\frac{3}{8} + \left(-\frac{2}{8}\right)$

$= -\frac{5}{8}$

d) $\frac{3}{8} + \left(-\frac{1}{4}\right) = \frac{3}{8} + \left(-\frac{2}{8}\right)$

$= \frac{1}{8}$

6. Add.

a) $\frac{2}{3} + \left(-1\frac{4}{11}\right) = \frac{2}{3} + \left(-\frac{15}{11}\right)$

$= \frac{22}{33} + \left(-\frac{45}{33}\right)$

$= -\frac{23}{33}$

b) $-1\frac{5}{6} + 3\frac{7}{8} = (-1 + 3) + \left(-\frac{5}{6} + \frac{7}{8}\right)$

$= 2 + \left(-\frac{20}{24} + \frac{21}{24}\right)$

$= 2\frac{1}{24}$

3.3 **7.** Find each difference.

a) $7.6 - 4.2 =$ **3.4**

b) $-3.4 - 5.7 =$ **−9.1**

c) $1.7 - (-9.3) =$ **11**

d) $-2.3 - (-5.6) =$ **3.3**

Estimate to check if your answers are reasonable.

8. Subtract.

a) $-\frac{5}{12} - \frac{1}{6} = -\frac{5}{12} + \left(-\frac{1}{6}\right)$

$= -\frac{5}{12} + \left(-\frac{2}{12}\right)$

$= -\frac{7}{12}$

b) $-2\frac{4}{7} - \left(-3\frac{1}{3}\right) = -2\frac{4}{7} + 3\frac{1}{3}$

$= -\frac{18}{7} + \frac{10}{3}$

$= -\frac{54}{21} + \frac{70}{21}$

$= \frac{16}{21}$

9. The table shows Lesley's temperature readings at different times one day.

Time	Temperature (°C)
9:00 A.M.	−5.4
12:00 P.M.	1.3
3:00 P.M.	2.7
9:00 P.M.	−4.2

Find the change in temperature between each pair of given times.
Did the temperature rise or fall each time?

a) 9:00 A.M. and 12:00 P.M.

Change in temperature: $1.3 - (-5.4)$

$= \mathbf{1.3} + \mathbf{5.4}$

$= \mathbf{6.7}$

The temperature **rose** by **6.7** °C.

b) 3:00 P.M. and 9:00 P.M.

Change in temperature: $\mathbf{-4.2} - \mathbf{2.7}$

$= \mathbf{-4.2} + \mathbf{(-2.7)}$

$= \mathbf{-6.9}$

The temperature **fell** by **6.9** °C.

c) 9:00 A.M. and 9:00 P.M.

Change in temperature: $\mathbf{-4.2 - (-5.4)}$

$= \mathbf{-4.2 + 5.4}$

$= \mathbf{1.2}$

The temperature rose by 1.2 degrees Celsius.

TEACHER NOTE

Next Steps: Direct students to *Practice* questions 4, 6, and 8 on page 121 of the Student Text.

3.4 Skill Builder

Writing a Fraction in Simplest Form

A fraction is in simplest form when the only common factor of the numerator and denominator is 1. For example, $\frac{5}{6}$ is in simplest form.

Writing a Fraction in Simplest Form

Look for common factors of the numerator and denominator.
Divide the numerator and denominator by common factors until you cannot go any further.

Write $\frac{24}{30}$ in simplest form.

*Factors of 24: 1, 2, 3, 4, **6**, 8, 12, 24*
*Factors of 30: 1, 2, 3, 5, **6**, 10, 15, 30*

Divide the numerator and the denominator by 6.

$$\frac{24}{30} \overset{\div 6}{=} \frac{4}{5}$$

$\frac{4}{5}$ is the simplest form of $\frac{24}{30}$.

TEACHER NOTE
Assign work selectively; not all students need explicit review of all 3 topics on pages 117 to 119.

Check

1. Write each fraction in simplest form.

TEACHER NOTE
Students may solve in one step, or multiple steps, depending on their choice of factor.

a) $\frac{10}{15} \overset{\div 5}{=} \frac{2}{3}$ Divide the numerator and the denominator by 5.

b) $\frac{14}{20} \overset{\div 2}{=} \frac{7}{10}$ Divide the numerator and the denominator by **2**.

c) $\frac{8}{12} \overset{\div 4}{=} \frac{2}{3}$ Divide the numerator and the denominator by **4**.

d) $\frac{12}{18} \overset{\div 6}{=} \frac{2}{3}$ Divide the numerator and the denominator by **6**.

Multiplying Proper Fractions

When multiplying fractions, we multiply the numerators, and we multiply the denominators.

$$\frac{2}{5} \times \frac{3}{8} = \frac{2 \times 3}{5 \times 8}$$
$$= \frac{6}{40}, \text{ or } \frac{3}{20}$$

To simplify, look for common factors *before* multiplying.

$$\frac{5}{12} \times \frac{8}{15} = \frac{5 \times 8}{12 \times 15}$$
$$= \frac{\cancel{5}^{1} \times \cancel{8}^{2}}{\cancel{12}^{3} \times \cancel{15}^{3}}$$
$$= \frac{1 \times 2}{3 \times 3}$$
$$= \frac{2}{9}$$

A common factor of 5 and 15 is 5.
A common factor of 8 and 12 is 4.
5 ÷ 5 = 1 *8 ÷ 4 = 2*
12 ÷ 4 = 3 *15 ÷ 5 = 3*

Check

1. Find each product.

a) $\frac{3}{4} \times \frac{2}{5}$

Multiply the numerators and multiply the denominators.

$= \frac{3 \times 2}{4 \times 5}$

A common factor of 2 and 4 is **2**.

$= \frac{3 \times \cancel{2}^{1}}{\cancel{4}^{2} \times 5}$

$= \frac{3 \times 1}{2 \times 5} = \frac{3}{10}$

b) $\frac{9}{14} \times \frac{7}{3}$

Multiply the numerators and multiply the denominators.

$= \frac{9 \times 7}{14 \times 3}$

A common factor of 9 and 3 is **3**.

A common factor of 7 and 14 is **7**.

$= \frac{\cancel{9}^{3} \times 7^{1}}{14^{2} \times \cancel{3}^{1}}$

$= \frac{3 \times 1}{2 \times 1} = \frac{3}{2}$

2. Multiply.

a) $\frac{6}{7} \times \frac{3}{4} = \frac{6 \times 3}{7 \times 4}$

$= \frac{\cancel{6}^{3} \times 3}{7 \times \cancel{4}^{2}}$

$= \frac{3 \times 3}{7 \times 2}$

$= \frac{9}{14}$

b) $\frac{4}{5} \times \frac{15}{14} = \frac{4 \times 15}{5 \times 14}$

$= \frac{\cancel{4}^{2} \times \cancel{15}^{3}}{\cancel{5}^{1} \times 14^{7}}$

$= \frac{2 \times 3}{1 \times 7}$

$= \frac{6}{7}$

c) $\frac{12}{5} \times \frac{5}{18} = \frac{12 \times 5}{5 \times 18}$

$= \frac{\cancel{12}^{2} \times \cancel{5}^{1}}{\cancel{5}^{1} \times \cancel{18}^{3}}$

$= \frac{2 \times 1}{1 \times 3}$

$= \frac{2}{3}$

Multiplying Mixed Numbers

Mixed numbers combine whole numbers with fraction parts.
To multiply, write the mixed numbers in fraction form.

Multiply: $2\frac{1}{4} \times \frac{2}{3}$

Rewrite $2\frac{1}{4}$ as an improper fraction: $2\frac{1}{4} = \frac{2 \times 4 + 1}{4}$

$= \frac{9}{4}$

So, $2\frac{1}{4} \times \frac{2}{3} = \frac{9}{4} \times \frac{2}{3}$ Multiply the numerators and multiply the denominators.

$= \frac{9 \times 2}{4 \times 3}$ Look for common factors in numerator and denominator.

$= \frac{\cancel{9}^{3} \times \cancel{2}^{1}}{\cancel{4}^{2} \times \cancel{3}^{1}}$

$= \frac{3}{2}$, or $1\frac{1}{2}$

Check

1. Write each mixed number as an improper fraction.

a) $3\frac{4}{5}$

$= \frac{3 \times 5 + 4}{5}$

$= \frac{19}{5}$

b) $3\frac{2}{7}$

$= \frac{3 \times 7 + 2}{7}$

$= \frac{23}{7}$

c) $1\frac{5}{12}$

$= \frac{1 \times 12 + 5}{12}$

$= \frac{17}{12}$

2. Multiply.

a) $3\frac{2}{5} \times \frac{1}{4}$ Rewrite $3\frac{2}{5}$ as an improper fraction: $3\frac{2}{5} = \frac{17}{5}$

$= \frac{17}{5} \times \frac{1}{4}$ Multiply the numerators and multiply the denominators.

$= \frac{17 \times 1}{5 \times 4}$

$= \frac{17}{20}$

b) $1\frac{1}{2} \times 1\frac{1}{3}$ Rewrite $1\frac{1}{2}$ and $1\frac{1}{3}$ as improper fractions.

$= \frac{3}{2} \times \frac{4}{3}$ Multiply the numerators and multiply the denominators.

$= \frac{3 \times 4}{2 \times 3}$ Look for common factors in numerator and denominator.

$= \frac{\cancel{3}^{1} \times \cancel{4}^{2}}{\cancel{2}^{1} \times \cancel{3}^{1}}$

$= 2$

3.4 Multiplying Rational Numbers

FOCUS **Multiply rational numbers.**

To predict the sign of the product of two rational numbers, use the sign rules for multiplying integers:

×	(−)	(+)
(−)	(+)	(−)
(+)	(−)	(+)

- *If the signs are the same, the answer is positive.*
- *If the signs are different, the answer is negative.*

Example 1 Multiplying Rational Numbers in Fraction Form

Multiply: $\left(-\frac{2}{3}\right)\left(-\frac{6}{7}\right)$

Solution

Predict the sign of the product:
Since the fractions have the same sign, their product is positive.

$$\left(-\frac{2}{3}\right)\left(-\frac{6}{7}\right) = \frac{(-2) \times (-6)^{-2}}{3^{1} \times 7}$$

$$= \frac{(-2) \times (-2)}{1 \times 7}$$

$$= \frac{4}{7}$$

So, $\left(-\frac{2}{3}\right)\left(-\frac{6}{7}\right) = \frac{4}{7}$

Check

1. Find each product.

a) $\frac{1}{5} \times \left(-\frac{3}{5}\right)$

The fractions have **different signs**, so their product is **negative**.

$$= \frac{\mathbf{1} \times (-3)}{\mathbf{5} \times \mathbf{5}}$$

$$= -\frac{\mathbf{3}}{\mathbf{25}}$$

b) $\left(-\frac{9}{11}\right)\left(-\frac{7}{12}\right)$

The fractions have **the same sign**, so their product is **positive**.

$= \frac{(-9) \times (-7)}{11 \times 12}$

A common factor of **9** and 12 is **3**.

$= \frac{(-9)^{-3} \times (-7)}{11 \times 12^{4}}$

$= \frac{(-3) \times (-7)}{11 \times 4}$

$= \frac{21}{44}$

Example 2 Multiplying Rational Numbers in Mixed Number Form

Multiply: $\left(-2\frac{1}{5}\right)\left(-1\frac{3}{4}\right)$

Solution

$\left(-2\frac{1}{5}\right)\left(-1\frac{3}{4}\right)$

Write each mixed number as an improper fraction.

$2\frac{1}{5} = \frac{10}{5} + \frac{1}{5} = \frac{11}{5}$ $\qquad$ $1\frac{3}{4} = \frac{4}{4} + \frac{3}{4} = \frac{7}{4}$

So, $\left(-2\frac{1}{5}\right)\left(-1\frac{3}{4}\right) = \left(-\frac{11}{5}\right)\left(-\frac{7}{4}\right)$

$= \frac{(-11) \times (-7)}{5 \times 4}$

$= \frac{77}{20}$, or $3\frac{17}{20}$

The numbers have the same sign: the product is positive.

$\frac{77}{20} = \frac{60}{20} + \frac{17}{20} = 3\frac{17}{20}$

Check

1. Find each product.

a) $\left(-1\frac{1}{4}\right) \times \frac{6}{7}$

$= \left(-\frac{5}{4}\right) \times \frac{6}{7}$

$= \frac{(-5) \times 6^{3}}{4^{2} \times 7}$

$= \frac{(-5) \times 3}{2 \times 7}$

$= -\frac{15}{14}$, or $-1\frac{1}{14}$

b) $\left(-2\frac{4}{5}\right)\left(-2\frac{3}{4}\right)$

$= \left(-\frac{14}{5}\right)\left(-\frac{11}{4}\right)$

$= \frac{(-14)^{-7} \times (-11)}{5 \times 4^{2}}$

$= \frac{(-7) \times (-11)}{5 \times 2}$

$= \frac{77}{10}$, or $7\frac{7}{10}$

To multiply rational numbers in decimal form:

- Use the sign rules for integers to find the sign of the product.
- Multiply as you would with whole numbers; estimate to place the decimal point.

Example 3 Multiplying Rational Numbers to Solve a Problem

On March 6, 2009, the price of a share in Bank of Montreal changed by −$3.05. Joanne owns 50 shares. By how much did the shares change in value that day?

Solution

The change in value is: $50 \times (-3.05)$

Multiply the integers, then estimate to place the decimal point.
$50 \times (-305) = -15\,250$

The product is negative.

Estimate to place the decimal point.
Since −3.05 is close to −3,
$50 \times (-3.05)$ is close to $50 \times (-3)$, or −150.
So, $50 \times (-3.05) = -152.50$

The shares changed in value by −$152.50 that day.

Check

1. On March 13, 2009, the price of a share in Research in Motion changed by −$1.13. Tania owns 80 shares. By how much did those shares change in value that day?

TEACHER NOTE
Some students' educational plans may recommend regular calculator use.

The change in value is: $80 \times (-1.13)$
The product is **negative**.
To find $80 \times (-1.13)$, multiply: **80** × **(−113)**
$80 \times$ **(−113)** = **−9040**
Estimate: $80 \times (-1.13)$ is about **80** × **(−1)** = **−80**
So, $80 \times (-1.13) =$ **−90.40**
The shares changed in value by **−$90.40** that day.

Practice

1. Is the product positive or negative?

a) $(-2.5) \times 3.6$ different signs; the product is **negative**.

b) $(-4.1) \times (-6.8)$ the same sign; the product is **positive**.

c) $\left(-\frac{3}{4}\right)\left(-\frac{7}{9}\right)$ **the same sign**; the product is **positive**.

d) $\left(-2\frac{1}{3}\right) \times 6\frac{1}{2}$ **different signs**; the product is **negative**.

2. Which of these expressions have the same product as $\frac{5}{8} \times \left(-\frac{7}{3}\right)$? Why?

a) $\left(-\frac{7}{3}\right) \times \frac{5}{8}$ **Yes**, since **changing the order of the factors does not change the product.**

b) $\left(-\frac{5}{8}\right)\left(-\frac{7}{3}\right)$ **No**, since **the product is positive, not negative.**

c) $\frac{7}{3} \times \frac{5}{8}$ **No**, since **the product is positive, not negative.**

d) $\frac{7}{3} \times \left(-\frac{5}{8}\right)$ **Yes**, since **the signs and numerical values match.**

3. Find each product.

Think: Is the product positive or negative?

a) $\frac{2}{7} \times \left(-\frac{5}{6}\right)$

$$\frac{2}{7} \times \left(-\frac{5}{6}\right) = \frac{2 \times (-5)}{7 \times 6}$$
$$= \frac{2^1 \times (-5)}{7 \times 6^3}$$
$$= \frac{1 \times (-5)}{7 \times 3}$$
$$= -\frac{5}{21}$$

b) $\left(-\frac{4}{5}\right)\left(-\frac{11}{12}\right)$

$$\left(-\frac{4}{5}\right)\left(-\frac{11}{12}\right) = \frac{(-4) \times (-11)}{5 \times 12}$$
$$= \frac{(-4)^{-1} \times (-11)}{5 \times 12^3}$$
$$= \frac{(-1) \times (-11)}{5 \times 3}$$
$$= \frac{11}{15}$$

4. Find each product.

a) $\left(-\frac{8}{9}\right) \times 1\frac{1}{2}$

$\left(-\frac{8}{9}\right) \times 1\frac{1}{2} = \left(-\frac{8}{9}\right) \times \frac{3}{2}$

$= \frac{(-8)^{-4} \times 3^{1}}{9^{3} \times 2^{1}}$

$= \frac{(-4) \times 1}{3 \times 1}$

$= -\frac{4}{3}$, or $-1\frac{1}{3}$

b) $\left(-2\frac{5}{6}\right)\left(-1\frac{1}{5}\right)$

$\left(-2\frac{5}{6}\right)\left(-1\frac{1}{5}\right) = \left(-\frac{17}{6}\right)\left(-\frac{6}{5}\right)$

$= \frac{(-17) \times (-6)^{-1}}{6^{1} \times 5}$

$= \frac{(-17) \times (-1)}{1 \times 5}$

$= \frac{17}{5}$

5. Multiply.

a) $0.4 \times (-3.2)$

To find $0.4 \times (-3.2)$, multiply: $4 \times (-32) =$ **−128**

$0.4 \times (-3.2)$ is about **1** $\times$ **(−3)** = **−3**

So, $0.4 \times (-3.2) =$ **−1.28**.

b) $(-3.03) \times (-0.7)$

To find $(-3.03) \times (-0.7)$, multiply: **(−303)** $\times$ **(−7)** = **2121**

$(-3.03) \times (-0.7)$ is about (**−3**) $\times$ (**−1**) = **3**

So, $(-3.03) \times (-0.7) =$ **2.121**.

6. On a certain day, the temperature changed by an average of −2.2°C/h.
What was the total temperature change in 8 h?

The total change in temperature is: **8** $\times$ **(−2.2)**

The product is **negative**.

To find **8 × (−2.2)**, multiply: **8** $\times$ **−22** = **−176**

$8 \times (-2.2)$ is about **8** $\times$ **(−2)** = **−16**.

So, $8 \times (-2.2) =$ **−17.6**

The temperature **fell** by **17.6** °C in 8 h.

TEACHER NOTE

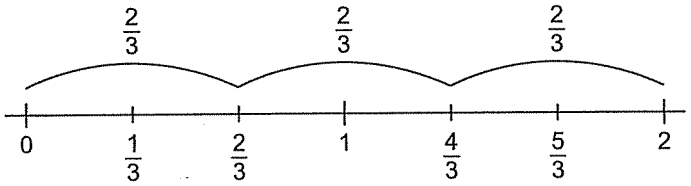
Next Steps: Direct students to questions 5, 7, 9, 10 on pages 127 to 128 of the Student Text.

3.5 Skill Builder

Dividing Fractions

Here are two ways to divide $2 \div \frac{2}{3}$.

- Use a number line.



How many groups of two-thirds are there in 2?

There are 3 groups of two-thirds in 2. So, $2 \div \frac{2}{3} = 3$

- Multiply by the reciprocal of $\frac{2}{3}$.

$2 \div \frac{2}{3}$ The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.

$= 2 \times \frac{3}{2}$

$= \frac{2}{1} \times \frac{3}{2}$

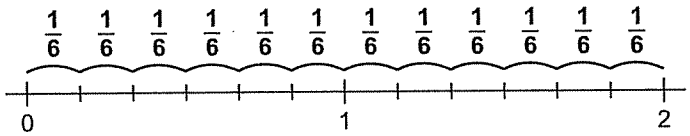
$= \frac{2^1 \times 3}{1 \times 2^1}$ Look for common factors.

$= 3$

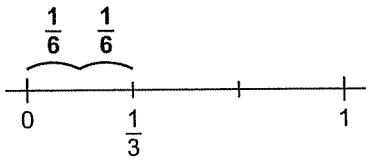
Check

1. Find each quotient. Use any method.

a) $2 \div \frac{1}{6} =$ **12**



b) $\frac{1}{3} \div 2 =$ **$\frac{1}{6}$**



c) $\frac{1}{3} \div \frac{5}{3} = \frac{1}{3} \times \frac{3}{5}$

$= \frac{1 \times 3^1}{3^1 \times 5}$

$= \frac{1}{5}$

d) $4 \div \frac{2}{3} = 4 \times \frac{3}{2}$

$= \frac{4^2 \times 3}{2^1}$

$= 6$

3.5 Dividing Rational Numbers

FOCUS **Divide rational numbers.**

Division is the opposite of multiplication.
So, the sign rules for dividing rational numbers are the same as those for multiplying rational numbers.

÷	(−)	(+)
(−)	(+)	(−)
(+)	(−)	(+)

Example 1 Dividing Rational Numbers in Fraction Form

Divide: $\frac{3}{4} \div \left(-\frac{9}{8}\right)$

Solution

$\frac{3}{4} \div \left(-\frac{9}{8}\right)$

The fractions have different signs, so the quotient is negative.

$\frac{3}{4} \div \left(-\frac{9}{8}\right) = \frac{3}{4} \times \left(-\frac{8}{9}\right)$ Multiply by the reciprocal.

$= \frac{3^1 \times (-8)^{-2}}{4^1 \times 9^3}$ Look for common factors.

$= \frac{1 \times (-2)}{1 \times 3}$

$= -\frac{2}{3}$

So, $\frac{3}{4} \div \left(-\frac{9}{8}\right) = -\frac{2}{3}$

Dividing by $-\frac{9}{8}$ is the same as multiplying by $-\frac{8}{9}$.

Check

1. Divide.

a) $\frac{2}{5} \div \left(-\frac{3}{4}\right)$

$= \frac{2}{5} \times \left(-\frac{4}{3}\right)$

$= \frac{2 \times (-4)}{5 \times 3}$

$= -\frac{8}{15}$

b) $\left(-\frac{2}{9}\right) \div \left(-\frac{4}{7}\right)$

$= \left(-\frac{2}{9}\right) \times \left(-\frac{7}{4}\right)$

$= \frac{(-2)^{-1} \times (-7)}{9 \times 4^2}$

$= \frac{(-1) \times (-7)}{9 \times 2}$

$= \frac{7}{18}$

Think: Is the quotient positive or negative?

TEACHER NOTE

For students experiencing success, introduce Example 1 part b on Student Text page 132. Assign *Practice* question 12 on page 135.

Example 2 Dividing Rational Numbers in Decimal Form

Divide:
$(-5.1) \div 3$

Solution

$(-5.1) \div 3$

Since the signs are different, the quotient is negative.

Divide integers: $(-51) \div 3 = -17$

Estimate to place the decimal point.
-5.1 is close to -6, so $(-5.1) \div 3$ is close to $(-6) \div 3 = -2$
So, $(-5.1) \div 3 = -1.7$

TEACHER NOTE
The elementary math curriculum restricts division to single-digit divisors. Allow calculator use, especially for solving applied problems.

Check

1. Divide: $(-7.5) \div 5$

$(-7.5) \div 5$

Divide integers: **(−75)** ÷ **5** = **−15**

Estimate to place the decimal point.

$(-7.5) \div 5$ is about **(−5)** ÷ **5** = **−1**

So, $-7.5 \div 5 =$ **−1.5**

Think: Is the quotient positive or negative?

Practice

1. Is the quotient positive or negative?

a) $(-7.5) \div (-3)$ Same sign; the quotient is **positive**.

b) $8.42 \div (-2)$ **Different signs**; the quotient is **negative**.

c) $\left(-\frac{9}{10}\right) \div \frac{3}{5}$ **Different signs**; the quotient is **negative**.

d) $(-16) \div \left(-\frac{4}{5}\right)$ **Same sign**; the quotient is **positive**.

2. Which of these expressions have the same answer as $\left(-\frac{3}{10}\right) \div \frac{2}{5}$?

a) $-\frac{3}{10} \times \frac{5}{2}$

Yes, since **the reciprocal of $\frac{2}{5}$ is $\frac{5}{2}$. To divide, you can multiply by the reciprocal.**

b) $-\frac{3}{10} \div \left(-\frac{2}{5}\right)$

No, since **the quotient is positive, not negative.**

c) $\frac{2}{5} \div \left(-\frac{3}{10}\right)$

No, since **changing the order of the factors changes the quotient.**

d) $\frac{3}{10} \div \left(-\frac{2}{5}\right)$

Yes, since **the sign and numerical value of the quotient are the same.**

3. Find each quotient.

a) $\left(-\frac{2}{3}\right) \div \frac{7}{6}$

$= \left(-\frac{2}{3}\right) \times \frac{6}{7}$

$= \frac{(-2) \times 6^2}{3^1 \times 7}$

$= \frac{(-2) \times 2}{1 \times 7}$

$= -\frac{4}{7}$

b) $\left(-\frac{15}{16}\right) \div \left(-\frac{5}{8}\right)$

$= \left(-\frac{15}{16}\right) \times \left(-\frac{8}{5}\right)$

$= \frac{(-15)^{-3} \times (-8)^{-1}}{16^2 \times 5^1}$

$= \frac{(-3) \times (-1)}{2 \times 1}$

$= \frac{3}{2}$, or $1\frac{1}{2}$

4. Divide.

a) $\left(-\frac{8}{9}\right) \div \frac{1}{3}$

$= \left(-\frac{8}{9}\right) \times \frac{3}{1}$

$= \frac{(-8) \times 3^{1}}{9^{3} \times 1}$

$= \frac{(-8) \times 1}{3 \times 1}$

$= -\frac{8}{3}$, or $-2\frac{2}{3}$

Think: Is the quotient positive or negative?

b) $\left(-\frac{2}{5}\right) \div \left(-\frac{3}{7}\right)$

$= \left(-\frac{2}{5}\right) \times \left(-\frac{7}{3}\right)$

$= \frac{(-2) \times (-7)}{5 \times 3}$

$= \frac{14}{15}$

5. Use integers to determine each quotient.
Estimate to place the decimal point in the answer.

a) $(-2.94) \div 0.7$

$(-2.94) \div 0.7$
The quotient is **negative**.
To find $(-2.94) \div 0.7$, divide: **(−294)** ÷ **7** = **−42**
$(-2.94) \div 0.7$ is about **(−3)** ÷ **1** = **−3**.
So, $(-2.94) \div 0.7$ = **−4.2**

b) $(-5.52) \div (-0.8)$

$(-5.52) \div (-0.8)$
The quotient is **positive**.
To find $(-5.52) \div (-0.8)$, divide: **(−552)** ÷ **(−8)** = **69**
$(-5.52) \div (-0.8)$ is about **(−6)** ÷ **(−1)** = **6**.
So, $(-5.52) \div (-0.8)$ = **6.9**

TEACHER NOTE
Next Steps: Direct students to questions 6, 11, 14, and 15 on pages 135 and 136 of the Student Text.

3.6 Order of Operations with Rational Numbers

The order of operations for rational numbers is the same as for integers and fractions.
Think BEDMAS to remember the correct order of operations.
We use this order of operations to evaluate expressions with more than one operation.

B Do the operations in brackets first.
E Next, evaluate any exponents.
D **M** Then, divide and multiply in order from left to right.
A **S** Finally, add and subtract in order from left to right.

Example 1 Using the Order of Operations with Decimals

Evaluate.

a) $(-2.4) \div 1.2 - 7 \times 0.2$ **b)** $(-3.4 + 0.6) + 4^2 \times 0.2$

Solution

a) $\mathbf{(-2.4)} \div \mathbf{1.2} - 7 \times 0.2$ Divide first.
$= -2 - \mathbf{7} \times \mathbf{0.2}$ Then multiply.
$= -2 - 1.4$ To subtract, add the opposite.
$= -2 + (-1.4)$
$= -3.4$

b) $\mathbf{(-3.4 + 0.6)} + 4^2 \times 0.2$ Brackets first.
$= -2.8 + \mathbf{4^2} \times 0.2$ Then evaluate the power.
$= -2.8 + \mathbf{16} \times \mathbf{0.2}$ Then multiply.
$= -2.8 + 3.2$ Add.
$= 0.4$

Check

1. Evaluate.

a) $3.8 + 0.8 \div (-0.2)$
$= 3.8 + (\underline{\mathbf{-4}})$
$= \underline{\mathbf{-0.2}}$

b) $4.6 - 3^2 + 3.9 \div (-1.3)$
$= 4.6 - \underline{\mathbf{9}} + 3.9 \div (-1.3)$
$= 4.6 - \underline{\mathbf{9}} + (\underline{\mathbf{-3}})$
$= -4.4 + (\underline{\mathbf{-3}})$
$= \underline{\mathbf{-7.4}}$

Example 2 Using the Order of Operations with Fractions

Evaluate:

a) $\left(\frac{3}{4}-\frac{7}{8}\right)\div\left(-\frac{5}{16}\right)$ **b)** $\left(-\frac{2}{3}\right)\times\frac{1}{6}+\frac{1}{2}$

Solution

a) $\left(\frac{3}{4}-\frac{7}{8}\right)\div\left(-\frac{5}{16}\right)$ Subtract in the brackets first. Use a common denominator of 8.

$=\left(\frac{6}{8}-\frac{7}{8}\right)\div\left(-\frac{5}{16}\right)$

$=\left(-\frac{1}{8}\right)\div\left(-\frac{5}{16}\right)$ To divide, multiply by the reciprocal of $-\frac{5}{16}$.

$=\left(-\frac{1}{8}\right)\times\left(-\frac{16}{5}\right)$

$=\left(-\frac{1}{\cancel{8}^{1}}\right)\times\left(-\frac{\cancel{16}^{2}}{5}\right)$ Look for common factors.

$=\frac{2}{5}$ Both factors are negative, so the product is positive.

b) $\left(-\frac{2}{3}\right)\times\frac{1}{6}+\frac{1}{2}$ Multiply first.

$=\left(-\frac{\cancel{2}^{1}}{3}\right)\times\frac{1}{\cancel{6}^{3}}+\frac{1}{2}$ Look for common factors.

$=\left(-\frac{1}{9}\right)+\frac{1}{2}$ Add. Use a common denominator of 18.

$=-\frac{2}{18}+\frac{9}{18}=\frac{7}{18}$

Check

1. Evaluate.

a) $\frac{3}{4}-\left(-\frac{2}{3}\right)\left(-\frac{1}{4}\right)$ Multiply first.

$=\frac{3}{4}-\frac{\cancel{(-2)}^{-1}\times(-1)}{3\times\cancel{4}^{2}}$ Look for common factors.

$=\frac{3}{4}-\frac{(-1)\times(-1)}{3\times 2}$

$=\frac{3}{4}-\frac{1}{6}$ Subtract. Use a common denominator of 12.

$=\frac{9}{12}-\frac{2}{12}$

$=\frac{7}{12}$

b) $\left(-\frac{1}{6}\right) \div \frac{1}{5} + \left(-\frac{3}{2}\right)$ Divide first. Multiply by the reciprocal of $\frac{\mathbf{1}}{\mathbf{5}}$.

$= -\frac{1}{6} \times \frac{\mathbf{5}}{\mathbf{1}} + \left(-\frac{3}{2}\right)$

$= \frac{\mathbf{(-1) \times 5}}{\mathbf{6 \times 1}} + \left(-\frac{3}{2}\right)$

$= \left(-\frac{\mathbf{5}}{\mathbf{6}}\right) + \left(-\frac{3}{2}\right)$ Add. Use a common denominator of **6**.

$= \left(-\frac{\mathbf{5}}{\mathbf{6}}\right) + \left(-\frac{\mathbf{9}}{\mathbf{6}}\right)$

$= \left(-\frac{\mathbf{14}}{\mathbf{6}}\right)$

Example 3 Applying the Order of Operations

The formula $C = (F - 32) \div 1.8$ converts temperatures in degrees Fahrenheit, F, to degrees Celsius, C.
What is 28.4°F in degrees Celsius?

Solution

Substitute $F = 28.4$ in the formula $C = (F - 32) \div 1.8$

$C = (28.4 - 32) \div 1.8$ Subtract in the brackets first. Add the opposite.

$= (28.4 + (-32)) \div 1.8$

$= (-3.6) \div 1.8$ Divide.

$= -2$

28.4°F is equivalent to −2°C.

Check

1. The expression $F = 32 + 9 \times C \div 5$ converts temperatures in degrees Celsius, C, to degrees Fahrenheit, F.

What is −12.5°C in degrees Fahrenheit?

$F = 32 + 9 \times (\mathbf{-12.5}) \div 5$ Multiply first.

$= \mathbf{32 + (-112.5) \div 5}$ **Then divide.**

$= \mathbf{32 + (-22.5)}$ **Then add.**

$= \mathbf{9.5}$

−12.5°C is equivalent to **9.5** °F.

Practice

1. In each expression, which operation will you do first?

a) $(-8.6) \times 2.4 - (-6 + 2.5)$

Add.

b) $2.5 - 6.4 \times 2.1 + 3.5$

Multiply.

c) $\frac{4}{3} \times \frac{5}{6} + \frac{2}{7} \div \frac{5}{14}$

Multiply.

d) $\frac{5}{3} + \frac{2}{7} \div \left(-\frac{1}{4}\right) - \frac{3}{5}$

Divide.

2. Evaluate each expression.

a) $(-3.6) \div 1.8 + (1.2 - 1.5)$

$= (-3.6) \div 1.8 + (-0.3)$

$= -2 + (-0.3)$

$= -2.3$

b) $\left(-\frac{1}{4}\right) \div \frac{3}{8} + \left(-\frac{1}{2}\right)^2$

$= \left(-\frac{1}{4}\right) \div \frac{3}{8} + \frac{1}{4}$

$= \left(-\frac{1}{4^1}\right) \times \frac{8^2}{3} + \frac{1}{4}$

$= -\frac{2}{3} + \frac{1}{4}$

$= -\frac{8}{12} + \frac{3}{12}$

$= -\frac{5}{12}$

3. Evaluate each expression.

a) $(5.6 + 4.4) \div (-2.5)$

$= 10 \div (-2.5)$

$= -4$

b) $(-4.2) + 6 \times (-1.7)$

$= (-4.2) + (-10.2)$

$= -14.4$

c) $9.2 \div 4 - 3.6 \times 2$

$= 2.3 - 3.6 \times 2$

$= 2.3 - 7.2$

$= -4.9$

d) $7.5 \times [-0.7 + (-0.3) \times 3]$

$= 7.5 \times [-0.7 - 0.9]$

$= 7.5 \times (-1.6)$

$= -12$

4. Evaluate each expression.

a) $\frac{1}{5} + \left(-\frac{1}{4}\right) \times \frac{8}{15}$

$= \frac{1}{5} + \frac{(-1)(8)}{4 \times 15}$

$= \frac{1}{5} + \frac{(-1) \times 8^{2}}{4^{1} \times 15}$

$= \frac{1}{5} + \left(-\frac{2}{15}\right)$

$= \left(\frac{3}{15}\right) + \left(-\frac{2}{15}\right)$

$= \frac{1}{15}$

b) $\left(-\frac{7}{4}\right) \div \frac{2}{3} + \frac{1}{4}$

$= \left(-\frac{7}{4}\right) \times \frac{3}{2} + \frac{1}{4}$

$= \frac{(-7) \times 3}{4 \times 2} + \frac{1}{4}$

$= -\frac{21}{8} + \frac{1}{4}$

$= -\frac{21}{8} + \frac{2}{8}$

$= -\frac{19}{8}$, or $-2\frac{3}{8}$

c) $\left(\frac{1}{3}\right)^{2} \times \frac{3}{2} - \frac{5}{4}$

$= \frac{1}{9} \times \frac{3}{2} - \frac{5}{4}$

$= \frac{1 \times 3^{1}}{9^{3} \times 2} - \frac{5}{4}$

$= \frac{1}{6} - \frac{5}{4}$

$= \frac{2}{12} - \frac{15}{12}$

$= -\frac{13}{12}$, or $-1\frac{1}{12}$

5. A mistake was made in each solution.
Identify the line in which the mistake was made, and give the correct solution.

a) $(-3.2 \div 1.6)^{2} - (-4.1)$

$= (-2)^{2} - (-4.1)$

$= 4 + (-4.1)$

$= -0.1$

Line 2:

4 − (−4.1) = 4 + (+4.1) = 8.1

b) $\frac{1}{3} + \frac{4}{3} \times \left(-\frac{1}{2}\right)$

$= \frac{5}{3} \times \left(-\frac{1}{2}\right)$

$= \frac{5 \times (-1)}{3 \times 2}$

$= -\frac{5}{6}$

Line 1:

$\frac{1}{3} + \frac{4^{2} \times (-1)}{3 \times 2^{1}}$

$= \frac{1}{3} + \left(-\frac{2}{3}\right)$

$= -\frac{1}{3}$

6. The formula for the area of a trapezoid is $A = h \times (a + b) \div 2$.
In the formula, h is the height and a and b are the lengths of the parallel sides. Find the area of a trapezoid with height 3.5 cm and parallel sides of length 8 cm and 12 cm.

Substitute $h =$ **3.5**, $a =$ **8**, and $b =$ **12** in the formula $A = h \times (a + b) \div 2$.

$A =$ **3.5 × (8 + 12) ÷ 2**

$=$ **3.5 × 20 ÷ 2**

$=$ **70 ÷ 2**

$=$ **35**

The trapezoid has area **35** cm^2.

TEACHER NOTE

Next Steps: Direct students to questions 6, 7, 8, and 11 on pages 140 to 141 of the Student Text.

Unit 3 Puzzle

Rational Numbers Bingo

Evaluate each expression and circle the answer on the Bingo cards. Which card is the winning card?

On the winning card, the answers form a horizontal, vertical, or diagonal line.

Questions

Evaluate as a decimal.

1. $(-8.2) - (-2.4) =$ **-5.8**

2. $3.65 \div (-0.5) =$ **-7.3**

3. $(-1.9) \times 2 =$ **-3.8**

4. $(-3.48) + 5.06 =$ **1.58**

5. $(-0.80) - 0.64 =$ **-1.44**

Evaluate as a fraction.

6. $\left(-\frac{7}{10}\right) + \frac{6}{5} =$ **$\frac{1}{2}$**

7. $\left(-\frac{6}{7}\right)\left(-\frac{14}{15}\right) =$ **$\frac{4}{5}$**

8. $\left(-\frac{1}{4}\right) \times \frac{1}{3} =$ **$-\frac{1}{12}$**

9. $\left(-\frac{4}{5}\right) - \left(-\frac{3}{4}\right) =$ **$-\frac{1}{20}$**

10. $\frac{1}{9} \div \left(-\frac{2}{3}\right) =$ **$-\frac{1}{6}$**

$-1\frac{11}{20}$	-0.16	$2\frac{1}{15}$	(−5.8)	$-2\frac{1}{12}$
7.3	−1	(−1.44)	$-\frac{4}{5}$	3.99
(−3.8)	$1\frac{2}{5}$	FREE SPACE	$1\frac{9}{10}$	$(-\frac{1}{12})$
3	$-2\frac{1}{15}$	−10.6	$(-\frac{1}{6})$	−1.58
(1.58)	$(\frac{4}{5})$	$(-\frac{1}{20})$	(−7.3)	$(\frac{1}{2})$

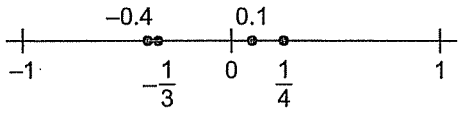
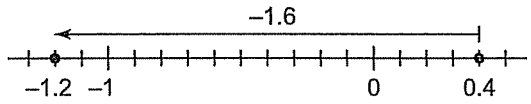
Card A

3	(−5.8)	−10.6	$1\frac{9}{10}$	$-2\frac{1}{15}$
$2\frac{1}{15}$	$-\frac{4}{5}$	−3.99	$1\frac{2}{5}$	(−7.3)
(−1.44)	(1.58)	FREE SPACE	$(-\frac{1}{6})$	−1
7.3	$7\frac{11}{20}$	$(\frac{1}{2})$	−1.58	$-2\frac{1}{12}$
$(-\frac{1}{20})$	3.99	$(\frac{4}{5})$	$(-\frac{1}{12})$	−0.16

Card B

The winning card is **Card A**.

Unit 3 Study Guide

Skill	Description	Example
Compare and order rational numbers.	Numbers increase in value from left to right on a number line.	 From least to greatest: $-0.4, -\frac{1}{3}, 0.1, \frac{1}{4}$
Add rational numbers.	Model on a number line: Start at the first number. Move right to add a positive number; move left to add a negative number.	 $0.4 + (-1.6) = -1.2$
	Look for common denominators to add fractions. With decimals, add digits with the same place value.	$-\frac{2}{5} + \frac{1}{2} = -\frac{4}{10} + \frac{5}{10} = \frac{1}{10}$ $(-18.7) + 13.5 = -5.2$
Subtract rational numbers.	Add the opposite.	$3\frac{1}{3} - \left(-1\frac{2}{5}\right) = 3\frac{1}{3} + \left(+1\frac{2}{5}\right)$ $= 3 + 1 + \frac{5}{15} + \frac{6}{15}$ $= 4\frac{11}{15}$ $-18.7 - 13.5 = -18.7 + (-13.5)$ $= -32.2$
Multiply and divide rational numbers.	Use the same rules for signs as with integers. Then determine the numerical value.	$\left(-\frac{2}{3}\right) \times \frac{9}{8} = \frac{(-2)^{-1} \times 3^3}{3^1 \times 8^4}$ $= -\frac{3}{4}$ $(-6.3) \times 7 = -44.1$
		$\left(-2\frac{1}{5}\right) \div \left(-3\frac{3}{10}\right) = \left(-\frac{11}{5}\right) \div \left(-\frac{33}{10}\right)$ $= \left(\frac{11^1}{5^1}\right) \times \left(-\frac{10^2}{33^3}\right)$ $= \frac{2}{3}$ $(-5.6) \div 0.7 = -8.0$
Use order of operations to evaluate expressions.	**B** Do the operations in brackets first. **E** Next, evaluate any exponents. **D** **M** Then, divide and multiply in order from left to right. **A** **S** Finally, add and subtract in order from left to right.	$(-2.50 + 1.75) \div (0.1 - (-0.4))^2$ $= -0.75 \div (0.1 + (+0.4))^2$ $= -0.75 \div (0.5)^2$ $= -0.75 \div 0.25$ $= -3$

Unit 3 Review

3.1 **1. a)** Write each number as a decimal.

i) $-\frac{16}{9} = \underline{-16 \div 9}$

$= \underline{-1.\overline{7}}$

ii) $-\frac{7}{3} = \underline{(-7) \div (3)}$

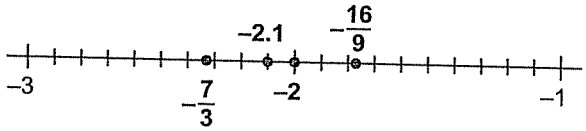
$= \underline{-2.\overline{3}}$

iii) $-2\frac{1}{5} = -\frac{11}{5}$

$= \underline{(-11) \div (5)}$

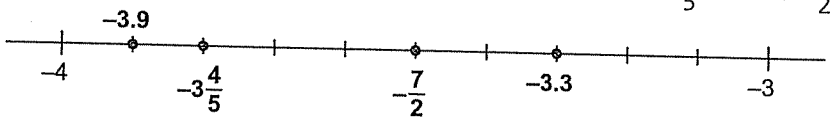
$= \underline{-2.2}$

b) Find two rational numbers between $-\frac{16}{9}$ and $-\frac{7}{3}$: **Sample answer:**



Two rational numbers between $-\frac{16}{9}$ and $-\frac{7}{3}$ are: **−2** and **−2.1**

2. Order these numbers from least to greatest: -3.9, $-3\frac{4}{5}$, -3.3, $-\frac{7}{2}$



From least to greatest: **$-3.9, -3\frac{4}{5}, -\frac{7}{2}, -3.3$**

3.2 **3.** Calculate each sum.

a) $(-2.1) + 4.8 = \underline{2.7}$

b) $25.6 + (-18.9) = \underline{6.7}$

c) $(-6.4) + (-3.8) = \underline{-10.2}$

4. Add.

a) $-\frac{1}{8} + \left(-\frac{3}{4}\right)$

$= -\frac{1}{8} + \underline{\left(-\frac{6}{8}\right)}$

$= \underline{-\frac{7}{8}}$

b) $-\frac{4}{3} + \frac{11}{12}$

$= \underline{-\frac{16}{12}} + \frac{11}{12}$

$= \underline{-\frac{5}{12}}$

c) $\left(-1\frac{2}{3}\right) + 2\frac{8}{9} = (-1 + 2) + \left(\underline{-\frac{2}{3}} + \underline{\frac{8}{9}}\right)$

$= (-1 + 2) + \left(\underline{-\frac{6}{9}} + \underline{\frac{8}{9}}\right)$

$= \underline{1 + \frac{2}{9}}$

$= \underline{1\frac{2}{9}}$

3.3 **5.** Subtract.

a) $\left(-\frac{7}{12}\right) - \left(-\frac{2}{3}\right) = -\frac{7}{12} + \frac{2}{3}$

$= -\frac{7}{12} + \frac{8}{12}$

$= \frac{1}{12}$

b) $\frac{3}{5} - 2\frac{1}{7} = \frac{3}{5} + \left(-\frac{15}{7}\right)$

$= \frac{21}{35} + \left(-\frac{75}{35}\right)$

$= -\frac{54}{35}$, or $-1\frac{19}{35}$

c) $-3\frac{1}{10} - 1\frac{3}{5} = -\frac{31}{10} + \left(-\frac{8}{5}\right)$

$= -\frac{31}{10} + \left(-\frac{16}{10}\right)$

$= -\frac{47}{10}$, or $-4\frac{7}{10}$

6. The table shows the elevations of several places on Earth.

Place	Elevation (m)
Mt. Everest	8849.7
Mt. Logan	5959.1
Death Valley	−410.9
Dead Sea	−417.3

Write a subtraction sentence that represents the difference in the elevations of the given locations. Then calculate the difference.

a) Mt. Logan and the Dead Sea

$5959.1 - (-417.3) = 5959.1 + 417.3$

$= 6376.4$

The difference in elevations is **6376.4** m.

b) Death Valley and the Dead Sea

$-410.9 - (-417.3) = -410.9 + 417.3$

$= 6.4$

The difference in elevations is **6.4** m.

c) Mt. Everest and Mt. Logan

$8849.7 - 5959.1 = 8849.7 + (-5959.1)$

$= 2890.6$

The difference in elevations is **2890.6** m.

3.4 **7.** What is the sign of each product?

a) $(-3.8) \times (-1.2)$ **Positive**

b) $0.75 \times (-8.6)$ **Negative**

c) $\left(-\frac{1}{3}\right)\left(-\frac{4}{9}\right)$ **Positive**

d) $\left(-1\frac{2}{5}\right) \times \frac{7}{10}$ **Negative**

8. Find each product.

a) $\left(-\frac{2}{5}\right)\left(-\frac{11}{20}\right)$

$= \frac{(-2)^{-1} \times (-11)}{5 \times 20^{10}}$

$= \frac{(-1) \times (-11)}{5 \times 10}$

$= \frac{11}{50}$

b) $\left(-\frac{4}{5}\right) \times \frac{25}{12}$

$= \frac{(-4)^{-1} \times 25^{5}}{5^{1} \times 12^{3}}$

$= \frac{(-1) \times 5}{1 \times 3}$

$= -\frac{5}{3}$, or $-1\frac{2}{3}$

c) $-\frac{15}{16} \times 1\frac{1}{3}$

$= -\frac{15}{16} \times \frac{4}{3}$

$= \frac{(-15)^{-5} \times 4^{1}}{16^{4} \times 3^{1}}$

$= \frac{(-5) \times 1}{4 \times 1}$

$= -\frac{5}{4}$, or $-1\frac{1}{4}$

d) $-3\frac{2}{3} \times \left(-2\frac{3}{11}\right)$

$= -\frac{11}{3} \times \left(-\frac{25}{11}\right)$

$= \frac{(-11)^{-1} \times (-25)}{3 \times 11^{1}}$

$= \frac{(-1) \times (-25)}{3 \times 1}$

$= \frac{25}{3}$, or $8\frac{1}{3}$

9. Circle the most reasonable answer.

	Question	Most reasonable answer		
a)	29.5 × 4.8	1.416	14.16	(141.6)
b)	5.4 × 0.7	0.378	(3.78)	37.8
c)	305.8 × 3.2	97.856	(978.56)	9785.6
d)	37.5 × 1.6	0.6	6	(60)

10. A diver descends at a speed of 0.8 m/min.
How far does the diver descend in 3.5 min?
The distance the diver descends is: **−0.8** × **3.5**

The product is **negative**. Multiply the whole numbers: **(−8)** × **35** = **−280**

Estimate: **−0.8** × **3.5** is about **(−1)** × **4** = **−4**.

The exact answer is **−0.8** × **3.5** = **−2.8**

The diver descends **2.8** m in 3.5 min.

3.5 **11.** Divide.

a) $\frac{1}{5} \div \left(-\frac{7}{10}\right)$

$= \frac{1}{5} \times \left(-\frac{10}{7}\right)$

$= \frac{1 \times (-10)^{-2}}{5^{1} \times 7}$

$= \frac{1 \times (-2)}{1 \times 7}$

$= -\frac{2}{7}$

b) $\left(-\frac{3}{5}\right) \div \left(-\frac{12}{7}\right)$

$= \left(-\frac{3}{5}\right) \times \left(-\frac{7}{12}\right)$

$= \frac{(-3)^{-1} \times (-7)}{5 \times 12^{4}}$

$= \frac{(-1) \times (-7)}{5 \times 4}$

$= \frac{7}{20}$

3.6 **12.** Evaluate each expression.

a) $1.1 - 3.1 \times 7$

$= 1.1 - 21.7$

$= 1.1 + (-21.7)$

$= -20.6$

b) $-1.8 \div (-0.3) + [5.1 - (-2.9)]$

$= -1.8 \div (-0.3) + [5.1 + 2.9]$

$= -1.8 \div (-0.3) + 8$

$= 6 + 8$

$= 14$

c) $\left(-\frac{5}{6}\right) \times \frac{1}{4} + \frac{5}{12}$

$= \frac{(-5) \times 1}{6 \times 4} + \frac{5}{12}$

$= -\frac{5}{24} + \frac{5}{12}$

$= -\frac{5}{24} + \frac{10}{24}$

$= \frac{5}{24}$

d) $1\frac{3}{4} + \frac{2}{3} \div \left(-\frac{8}{9}\right)$

$= 1\frac{3}{4} + \frac{2}{3} \times \left(-\frac{9}{8}\right)$

$= 1\frac{3}{4} + \frac{2^{1} \times (-9)^{-3}}{3^{1} \times 8^{4}}$

$= 1\frac{3}{4} + \left(-\frac{3}{4}\right)$

$= \frac{7}{4} + \left(-\frac{3}{4}\right)$

$= \frac{4}{4}$, or 1

Linear Relations

What You'll Learn

- Use expressions and equations to write patterns and work with them.
- Use substitution to work with patterns to find more information.
- Graph and analyze linear relations.
- Use interpolation and extrapolation to gather more information from graphs.

Why It's Important

Patterns and linear relations are used by

- book printers, to quote the cost of a job
- managers, to plan for new hiring

Key Words

variable
expression
equation
table of values
relation
linear relation
coordinates
coordinate grid
discrete
origin
vertical
horizontal
oblique
interpolation
extrapolation

11. Divide.

a) $\frac{16a-40}{8}$

$= \frac{16a}{8} + \frac{-40}{8}$

$= \frac{16}{8} \times a + (-5)$

$= 2 \times a - 5$

$= 2a - 5$

b) $\frac{27b^2 - 9b + 36}{-9}$

$= \frac{27b^2}{-9} + \frac{-9b}{-9} + \frac{36}{-9}$

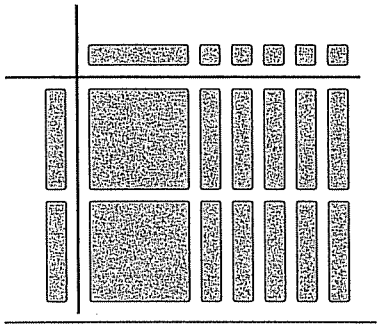
$= \frac{27}{-9} \times b^2 + \frac{-9}{-9} \times b + (-4)$

$= (-3) \times b^2 + 1 \times b - 4$

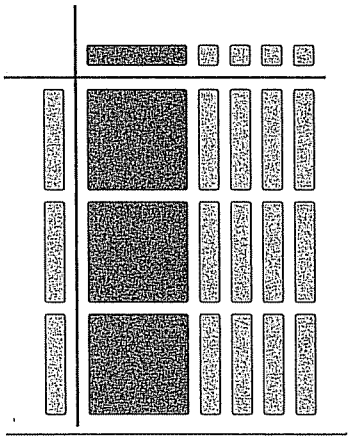
$= -3b^2 + b - 4$

5.6 **12.** Sketch algebra tiles to multiply. Write the product each time.

a) $2c(c + 5) = 2c^2 + 10c$



b) $3d(-d + 4) = -3d^2 + 12d$



13. Multiply.

a) $3e(5e - 2)$

$= (3e)(5e) + (3e)(-2)$

$= 15e^2 + (-6)e$

$= 15e^2 - 6e$

b) $-4f(5f + 2)$

$= (-4f)(5f) + (-4f)(2)$

$= -20f^2 + (-8f)$

$= -20f^2 - 8f$

TEACHER NOTE

Next Steps: Direct students to questions 2, 3, 5, 6, 8, 9, 10, 14, 17, 22, and 24 on pages 259, 260, and 261 of the Student Text.

14. Divide.

a) $\frac{-21k^2}{7k}$

$= \frac{-21}{7} \times \frac{k^2}{k}$

$= -3 \times \frac{k \times k^1}{k_1}$

$= -3 \times k$

$= -3k$

b) $\frac{81m^2 - 45m}{-9m}$

$= \frac{81m^2}{-9m} + \frac{-45m}{-9m}$

$= \frac{81}{-9} \times \frac{m^2}{m} + \frac{-45}{-9} \times \frac{m}{m}$

$= -9 \times m + 5 \times 1$

$= -9m + 5$

c) $\frac{-33n^2 + 36n}{-3n}$

$= \frac{-33n^2}{-3n} + \frac{36n}{-3n}$

$= \frac{-33}{-3} \times \frac{n^2}{n} + \frac{36}{-3} \times \frac{n}{n}$

$= 11 \times n + (-12) \times 1$

$= 11n - 12$