UNIT 6

Linear Equations and Inequalities

What You'll Learn

- Expand your understanding of solving equations.
- Model and solve problems using linear equations.
- Investigate the properties of inequalities.
- Explain and illustrate strategies to solve linear inequalities.

Why It's Important

Linear equations and inequalities are used by

- nurses, home health aides, and medical assistants, to take temperatures and blood pressures, and set up equipment
- purchasing agents and buyers, to find the best merchandise at the lowest price for their employers, and stay aware of changes in the marketplace

Key Words

inverse operations
variable
inequality

6.1 Skill Builder

Order of Operations

We use this order of operations to evaluate expressions with more than one operation.

TEACHER NOTE
For related review, see Unit 2, Lesson 2.3, and Unit 4, Lessons 4.1 and 4.3.

B Do the operations in **b**rackets first
E Evaluate any **e**xponents
D
M **D**ivide and **m**ultiply in order from left to right
A
S **A**dd and **s**ubtract in order from left to right

$7 - 8 \div 2 + (6 - 1)$	Evaluate brackets first: $(6 - 1)$
$= 7 - 8 \div 2 + 5$	Then divide: $8 \div 2$
$= 7 - 4 + 5$	Then add and subtract from left to right.
$= 3 + 5$	
$= 8$	

Check

1. In each expression, circle what you will do first.

a) $-7 + 2 \times (-3)$ Add (Multiply)

b) $3 \times (-10 \div 2) - (-4)$ Multiply (Divide) Subtract

c) $19 - 4 \times 3^2 \div 6$ Subtract Multiply (Power) Divide

d) $-30 \div 5 - 10 \times 2$ (Divide) Subtract Multiply

2. Evaluate.

a) $-17 + 4 \times 3$
$= -17 +$ **12**
$=$ **−5**

b) $-16 \div 4 + 24 \div (-8)$
$=$ **−4** $+ 24 \div (-8)$
$=$ **−4** $+ ($**−3**$)$
$=$ **−7**

c) $3^2 + 4^2 \div 8 + (-5)$
$=$ **9** $+$ **16** $\div 8 + (-5)$
$= 9 +$ **2** $+ (-5)$
$=$ **11** $+ (-5)$
$=$ **6**

The Distributive Property

To multiply $5 \times (3 + 4)$, we can:

- Add $3 + 4$ in the brackets, then multiply the sum by 5:
  $5 \times (3 + 4)$
  $= 5 \times 7$
  $= 35$

OR

- Multiply each number in the brackets by 5, then add:
  $5 \times (3 + 4) = 5 \times 3 + 5 \times 4$
  $= 15 + 20$
  $= 35$

We can use the distributive property to write this expression as a sum of terms:
$7(a + b) = 7a + 7b$

Check

1. Expand.

a) $3(b - 2)$
$= 3b - 3(2)$
$= 3b - 6$

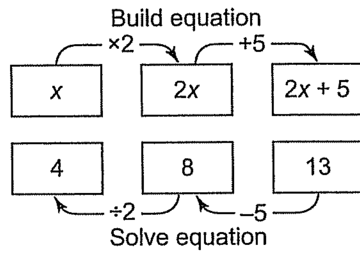
b) $6(2 - y)$
$= 6(2) - 6(y)$
$= 12 - 6y$

TEACHER NOTE
For related review, see *Math Makes Sense* 8, Sections 6.4 and 6.5.

6.1 Solving Equations by Using Inverse Operations

FOCUS **Model a problem with a linear equation, and solve the equation pictorially and symbolically.**

Look at the equation $2x + 5 = 13$.
How was it built?
Start with x. Multiply by 2, then add 5.



To solve the equation, "undo" the operations, in reverse order.

$2x + 5 = 13$ Subtract 5.
$2x + 5 - 5 = 13 - 5$
$2x = 8$ Divide by 2.
$x = 4$

Do the same operation to both sides of the equation to preserve the equality.

Inverse operations undo each other's results. For example: addition and subtraction are inverse operations.

Example 1 Writing Then Solving One-Step Equations

Write then solve an equation to find each number. Verify the solution.

a) A number plus 5 is 20. **b)** Four times a number is −32.

Solution

a) Let x represent the number. Then, x plus 5 is 20.
The equation is: $x + 5 = 20$
To solve the equation, apply the inverse operations.

$x + 5 = 20$ Undo the addition. Subtract 5 from each side.
$x + 5 - 5 = 20 - 5$
$x = 15$

To verify the solution, substitute $x = 15$ into $x + 5 = 20$.
$15 + 5 = 20$, so the solution is correct.

b) Let n represent the number. Then, 4 times n is −32.
The equation is: $4n = -32$
To solve the equation, apply the inverse operations.

$4n = -32$ Undo the multiplication. Divide each side by 4.
$\frac{4n}{4} = \frac{-32}{4}$
$n = -8$

To verify the solution, substitute $n = -8$ into $4n = -32$.
$4(-8) = -32$, so the solution is correct.

Check

1. Let n represent a number. Two less than a number is 10. What is the number?

$n - \underline{2} = \underline{10}$

$n - \underline{2 + 2} = \underline{10 + 2}$

$n = \underline{12}$

Check: $\underline{12 - 2 = 10}$

Example 2 Solving a Two-Step Equation

Solve, then verify each equation.

a) $3x + 4 = -5$

b) $2(-2 + w) = 18$

Solution

a) Perform the inverse operations in reverse order.

$3x + 4 = -5$ Subtract 4 from each side.

$3x + 4 - 4 = -5 - 4$

$3x = -9$ Divide each side by 3.

$\frac{3x}{3} = \frac{-9}{3}$

$x = -3$

To verify the solution, substitute $x = -3$ into $3x + 4 = -5$.

Left side $= 3x + 4$ Right side $= -5$

$= 3(-3) + 4$

$= -9 + 4$

$= -5$

Since the left side equals the right side, $x = -3$ is correct.

b) $2(-2 + w) = 18$ Use the distributive property to expand $2(-2 + w)$.

$2(-2) + 2(w) = 18$

$-4 + 2w = 18$ Add 4 to each side.

$-4 + 2w + 4 = 18 + 4$

$2w = 22$ Divide each side by 2.

$\frac{2w}{2} = \frac{22}{2}$

$w = 11$

To verify the solution, substitute $w = 11$ into $2(-2 + w) = 18$.

Left side $= 2(-2 + w)$ Right side $= 18$

$= 2(-2 + 11)$

$= 2(9)$

$= 18$

Since the left side equals the right side, $w = 11$ is correct.

Check

1. What operations would you use to solve each equation?

a) $-5h + 4 = 6$

First **subtract 4**, then **divide by −5**.

b) $2 + 5p = -3$

First **subtract 2**, then **divide by 5**.

2. Solve, then verify the equation.

$2(t - 1) = 12$

$2(\underline{t}) - 2(\underline{1}) = 12$

$\mathbf{2t - 2 = 12}$

$\mathbf{2t - 2 + 2 = 12 + 2}$

$\mathbf{2t = 14}$

$\mathbf{\frac{2t}{2} = \frac{14}{2}}$

$\mathbf{t = 7}$

Use the distributive property to expand 2(t − 1).

Substitute $t =$ **7** into the equation.

Left side $= 2(t - 1)$ Right side = **12**

$= 2(\mathbf{7 - 1})$

$= \mathbf{2(6)}$

$= \mathbf{12}$

Since the left side equals the right side, $t =$ **7** is correct.

Practice

1. Solve each equation.

a) $z + 9 = 10$

$\mathbf{z + 9 - 9 = 10 - 9}$

$\mathbf{z = 1}$

b) $s - 4 = -12$

$\mathbf{s - 4 + 4 = -12 + 4}$

$\mathbf{s = -8}$

c) $6 + c = 2$

$\mathbf{6 + c - 6 = 2 - 6}$

$\mathbf{c = -4}$

d) $5 = v - 2$

$\mathbf{5 + 2 = v - 2 + 2}$

$\mathbf{7 = v}$

2. For each statement, write then solve an equation to find the number. Verify the solution.

a) A number divided by 4 is −3.

$\frac{n}{4} = -3$

$4 \times \frac{n}{4} = 4(-3)$

$n = -12$

Left side $= \frac{n}{4}$

$= \frac{-12}{4}$

$= -3$

Right side $= -3$

$n = -12$ is correct.

b) Three times a number is 15.

$3x = 15$

$\frac{3x}{3} = \frac{15}{3}$

$x = 5$

Left side $= 3x$

$= 3(5)$

$= 15$

Right side $= 15$

$x = 5$ is correct.

3. Emma tried to solve the equation $4x = 16$ by subtracting 4 from each side. Show the correct way to solve the equation.

$4x = 16$

$\frac{4x}{4} = \frac{16}{4}$

$x = 4$

4. Solve each equation. Verify the solution.

a) $5k - 6 = 24$

$5k - 6 + 6 = 24 + 6$

$5k = 30$

$\frac{5k}{5} = \frac{30}{5}$

$k = 6$

Left side $= 5k - 6$

$= 5(6) - 6$

$= 30 - 6$

$= 24$

Right side $= 24$

$k = 6$ is correct.

b) $3 + 4y = -9$

$3 + 4y - 3 = -9 - 3$

$4y = -12$

$\frac{4y}{4} = \frac{-12}{4}$

$y = -3$

Left side $= 3 + 4y$

$= 3 + 4(-3)$

$= 3 - 12$

$= -9$

Right side $= -9$

$y = -3$ is correct.

5. a) Tuyen tried to solve the equation $3x - 6 = 15$ like this:

$$\frac{3x}{3} - 6 = \frac{15}{3}$$
$$x - 6 = 5$$
$$x - 6 + 6 = 5 + 6$$
$$x = 11$$

Where did she make a mistake?

She divided by 3 instead of adding 6 first.

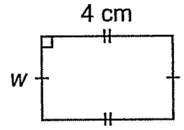
b) Show the correct way to solve $3x - 6 = 15$.
Verify the solution.

$$\mathbf{3x - 6 = 15}$$
$$\mathbf{3x - 6 + 6 = 15 + 6}$$
$$\mathbf{3x = 21}$$
$$\mathbf{\frac{3x}{3} = \frac{21}{3}}$$
$$\mathbf{x = 7}$$

Left side $= 3x - 6$ Right side $=$ **15**
$= \mathbf{3(7) - 6}$
$= \mathbf{21 - 6}$
$= \mathbf{15}$

Since the left side equals the right side, $x =$ **7** is correct.

6. A rectangle has length 4 cm and perimeter 12 cm.



The perimeter is the sum of all the sides.

a) Write an equation that can be used to determine the width of the rectangle.
$\mathbf{4 + 4 + w + w} = 12$

b) Solve the equation.
$\mathbf{8 + 2w = 12}$
$\mathbf{8 + 2w - 8 = 12 - 8}$
$\mathbf{2w = 4}$
$\mathbf{w = 2}$
The width is **2** cm.

c) Verify the solution.
Left side $= \mathbf{8 + 2w}$ Right side $=$ **12**
$\mathbf{= 8 + 2(2)}$
$\mathbf{= 8 + 4}$
$\mathbf{= 12}$
Since the left side equals the right side, $w = 2$ is correct.

TEACHER NOTE

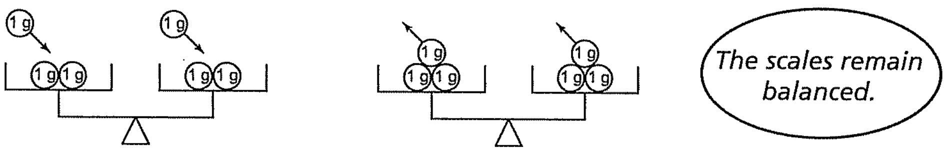
Next Steps: Direct students to questions 7, 9a, 10a, d, and 11 a, b on pages 272 and 273 of the Student Text.

For students experiencing success, introduce Example 3 on page 270 of the Student Text, and assign Practice questions 14 and 18.

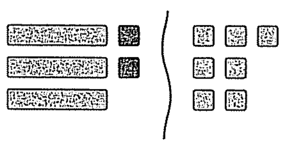
6.2 Skill Builder

Solving Equations Using Models

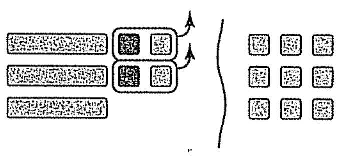
We can use a balance-scales model to solve an equation.
Keep the scales balanced by doing the same operation on both sides.
For example, we can add or remove the same mass:



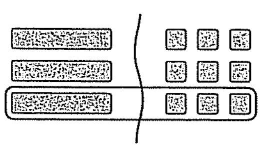
We can also use algebra tiles to solve an equation.
Rearrange the tiles so the variable tiles are on one side,
and the unit tiles are on the other side.
For example, to solve $3x - 2 = 7$:



Isolate the x-tiles.
Add 2 positive unit tiles to make zero pairs.



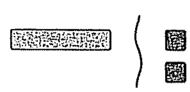
There are 3 x-tiles.
Arrange the unit tiles into 3 equal groups.



The solution is $x = 3$.

Check

1. Use algebra tiles to solve: $4m + 6 = -2$
Record your steps algebraically.



$4m + 6 - 6 = -2 - 6$

$4m = -8$

$\frac{4m}{4} = \frac{-8}{4}$

$m = -2$

TEACHER NOTE
For related review, see *Math Makes Sense 8*, Section 6.1.

6.2 Solving Equations by Using Balance Strategies

FOCUS **Model a problem with a linear equation, use balance strategies to solve the equation pictorially, and record the process symbolically.**

To solve an equation, isolate the variable on one side of the equation.

We can use balance scales to model this.

Everything we do to one side of the equation must be done to the other side.
This way, the scales remain balanced.

Example 1 Modelling Equations with Variables on Both Sides

a) Solve: $3x + 2 = 6 + x$

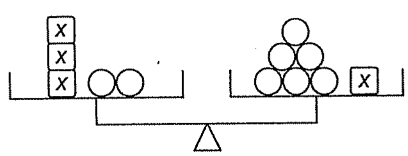
b) Verify the solution.

Solution

a) Isolate x to solve the equation.

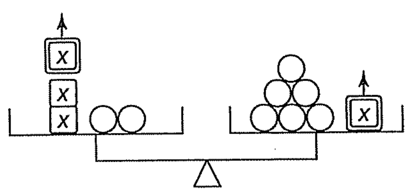
There can be more than one way to solve an equation.

Pictorial Solution | **Algebraic Solution**

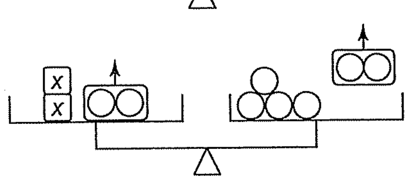


$$3x + 2 = 6 + x$$

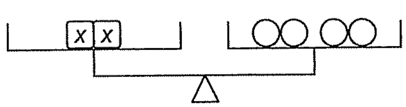
Each ◯ has a mass of 1 g.



$$3x + 2 - x = 6 + x - x$$
$$2x + 2 = 6$$



$$2x + 2 - 2 = 6 - 2$$
$$2x = 4$$



$$\frac{2x}{2} = \frac{4}{2}$$
$$x = 2$$

b) Check: Substitute $x = 2$ in each side of the equation.

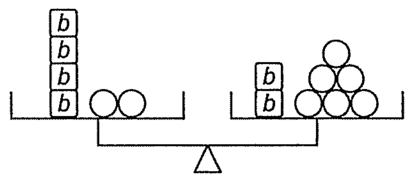
Left side $= 3x + 2$
$= 3(2) + 2$
$= 6 + 2$
$= 8$

Right side $= 6 + x$
$= 6 + 2$
$= 8$

Since the left side equals the right side, $x = 2$ is correct.

Check

1. Write the equation given by the picture.
Solve, and record your steps algebraically.



$4b + \underline{\mathbf{2}} = \underline{\mathbf{2b}} + \underline{\mathbf{6}}$

$4b + \underline{\mathbf{2}} - \underline{\mathbf{2b}} = \underline{\mathbf{2b}} + \underline{\mathbf{6}} - \underline{\mathbf{2b}}$

$\mathbf{2b + 2 = 6}$

$\mathbf{2b + 2 - 2 = 6 - 2}$

$\mathbf{2b = 4}$

$\mathbf{\frac{2b}{2} = \frac{4}{2}}$

$\mathbf{b = 2}$

If we have an equation with negative terms, it is easier to use algebra tiles to model and solve the equation.

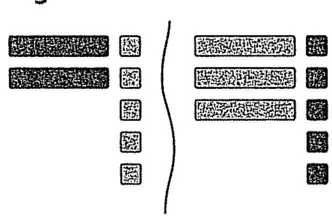
We add the same tiles to each side or subtract the same tiles from each side to keep the equation balanced.

Example 2 Using Algebra Tiles to Solve an Equation

Solve: $-2n + 5 = 3n - 5$

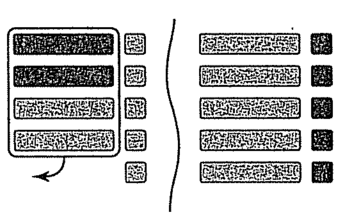
Solution

Algebra Tile Model | **Algebraic Solution**



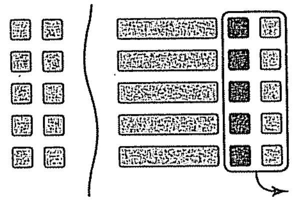
$-2n + 5 = 3n - 5$

Add 2 n-tiles to each side.
Remove zero pairs.



$-2n + 5 + 2n = 3n - 5 + 2n$
$5 = 5n - 5$

Add five 1-tiles to each side.
Remove zero pairs.

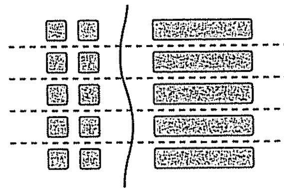


$5 + 5 = 5n - 5 + 5$

$10 = 5n$

You can solve with the variable on either side of the equal sign. The answer is the same.

Arrange the remaining tiles on each side into 5 groups. One n-tile is equal to 2.



$\frac{10}{5} = \frac{5n}{5}$

$2 = n$

Or, $n = 2$

Check

1. Use algebra tiles to model and solve the equation.
Record your work algebraically.

$$-c + 5 = 2c - 4$$

$$-c + 5 - \underline{\ \mathbf{2}\ }c = 2c - 4 - \underline{\ \mathbf{2}\ }c$$

$$\underline{\mathbf{-3}}\,c + 5 = \underline{\mathbf{-4}}$$

$$\underline{\mathbf{-3}}\,c + 5 - \underline{\ \mathbf{5}\ } = \underline{\mathbf{-4}} - \underline{\ \mathbf{5}\ }$$

$$\underline{\mathbf{-3}}\,c = \underline{\mathbf{-9}}$$

$$\frac{\mathbf{-3c}}{\mathbf{-3}} = \frac{\mathbf{-9}}{\mathbf{-3}}$$

$$c = \underline{\ \mathbf{3}\ }$$

Example 3 Solving Equations with Rational Coefficients

Solve the equation, then verify the solution.

$\frac{2a}{3} = 6$

Solution

Create an equivalent equation without fractions.
To clear the fraction, multiply each side by the denominator.

$\frac{2a}{3} = 6$	Multiply each side by 3.
$\frac{2a}{3} \times 3 = 6 \times 3$	$\frac{2a}{3} \times \frac{3}{1} = \frac{2a}{1} = 2a$
$2a = 18$	Divide each side by 2.
$\frac{2a}{2} = \frac{18}{2}$	
$a = 9$	

Check: Substitute $a = 9$ in $\frac{2a}{3} = 6$.

Left side $= \frac{2a}{3}$ Right side $= 6$

$= \frac{2(9)}{3}$

$= \frac{18}{3}$

$= 6$

Since the left side equals the right side, $a = 9$ is correct.

Check

1. Solve. Verify the solution.

a) $\frac{x}{4} = 5$

$4 \times \frac{x}{4} = 4(5)$

$x = 20$

Clear the fraction. Multiply each side by the denominator, 4.

Check: Substitute $x =$ **20** in $\frac{x}{4} = 5$.

Left side $= \frac{20}{4}$ Right side $=$ **5**

$= 5$

Since **the left side equals the right side,** $x =$ **20** is correct.

b) $\frac{x}{4} + \frac{7}{4} = \frac{5}{4}$

$4 \times \frac{x}{4} + 4 \times \frac{7}{4} = 4 \times \frac{5}{4}$

$x + 7 = 5$

$x + 7 - 7 = 5 - 7$

$x = -2$

Check: Substitute $x =$ **−2** in $\frac{x}{4} + \frac{7}{4} = \frac{5}{4}$.

Left side $= \frac{-2}{4} + \frac{7}{4}$ Right side $= \frac{5}{4}$

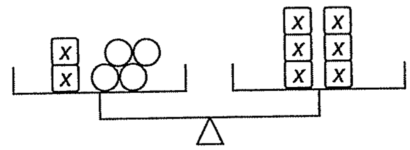
$= \frac{-2 + 7}{4}$

$= \frac{5}{4}$

Since **the left side equals the right side,** $x =$ **−2** is correct.

Practice

1. Write the equation represented by the picture.
Solve, and record your steps algebraically.



$2x + 4 = 6x$

$2x + 4 - 2x = 6x - 2x$

$4 = 4x$

$\frac{4}{4} = \frac{4x}{4}$

$x = 1$

2. Solve each equation.

a) $3w - 2 = w + 4$

$3w - 2 \underline{+ 2} = w + 4 \underline{+ 2}$

$3w = w + 6$

$3w - w = w + 6 - w$

$2w = 6$

$\frac{2w}{2} = \frac{6}{2}$

$w = 3$

b) $2 - x = -2 - 3x$

$2 - x \underline{+ 3x} = -2 - 3x \underline{+ 3x}$

$2 + 2x = -2$

$2 + 2x - 2 = -2 - 2$

$2x = -4$

$\frac{2x}{2} = \frac{-4}{2}$

$x = -2$

c) $y - 4 = -2 - y$

$y - 4 + 4 = -2 - y + 4$

$y = 2 - y$

$y + y = 2 - y + y$

$2y = 2$

$\frac{2y}{2} = \frac{2}{2}$

$y = 1$

d) $2 - j = -8 + 4j$

$2 - j + j = -8 + 4j + j$

$2 = -8 + 5j$

$2 + 8 = -8 + 5j + 8$

$10 = 5j$

$\frac{10}{5} = \frac{5j}{5}$

$2 = j$

3. Solve each equation. Verify the solution.

a) $\frac{t}{6} + 2 = 4$

$\frac{t}{6} + 2 - \underline{2} = 4 - \underline{2}$

$\frac{t}{6} = \underline{2}$

$6 \times \frac{t}{6} = 6(2)$

$t = 12$

Left side $= \frac{t}{6} + 2$

$= \frac{12}{6} + 2$

$= 2 + 2$

$= 4$

Right side $= \underline{4}$

$t = \underline{12}$ is correct.

b) $5 + \frac{w}{5} = 2$

$5 + \frac{w}{5} - 5 = 2 - 5$

$\frac{w}{5} = -3$

$5 \times \frac{w}{5} = 5(-3)$

$w = -15$

Left side $= 5 + \frac{w}{5}$

$= $ **$5 + \frac{-15}{5}$**

$=$ **$5 + (-3)$**

$=$ **2**

Right side $=$ **2**

$w =$ **-15** is correct.

4. Jake tried to solve $4c - 3 = c + 3$ like this:

$$4c - 3 - 3 = c + 3 - 3$$
$$4c = c + 0$$
$$4c - c = c - c + 0$$
$$3c = 0$$
$$c = 0$$

a) Where did he make a mistake?

He forgot to write that $-3 - 3$ is -6 in the second line.

b) Show the correct way to solve $4c - 3 = c + 3$.
Verify the solution.

$4c - 3 = c + 3$

$4c - 3 + 3 = c + 3 + 3$

$4c = c + 6$

$4c - c = c + 6 - c$

$3c = 6$

$\frac{3c}{3} = \frac{6}{3}$

$c = 2$

Left side $= 4c - 3$

$= 4(2) - 3$

$= 8 - 3$

$= 5$

Right side $= c + 3$

$= 2 + 3$

$= 5$

Since the left side equals the right side, $c =$ **2** is correct.

TEACHER NOTE

Next Steps: Direct students to questions 6, 7, 9, 10, and 11 on page 281 of the Student Text.

For students experiencing success, introduce Example 4 on page 279 of the Student Text, and assign Practice questions 12–14.

CHECKPOINT

Can you ...

- Model a problem with a linear equation, and solve the equation pictorially and symbolically?
- Model a problem with a linear equation, use balance strategies to solve the equation pictorially, and record the process symbolically?

6.1

1. For each equation, what is the first operation you would do to isolate the variable?

a) $3k = 9$
Divide by 3.

b) $m - 2 = 5$
Add 2.

c) $2x - 3 = 4$
Add 3.

d) $5 = 3y - 4$
Add 4.

2. For each statement, write then solve an equation to find the number. Verify the solution.

a) Two times a number is 10.

$2x = 10$

$\frac{2x}{2} = \frac{10}{2}$

$x = 5$

$2(5) = 10$, so the solution is correct.

b) Three less than a number is 15.

$y - 3 = 15$

$y - 3 + 3 = 15 + 3$

$y = 18$

$18 - 3 = 15$, so the solution is correct.

3. Solve each equation.

a) $x + 7 = -2$

$x + 7 - 7 = -2 - 7$

$x = -9$

b) $4c = 20$

$\frac{4c}{4} = \frac{20}{4}$

$c = 5$

c) $4 = y - 2$

$4 + 2 = y - 2 + 2$

$6 = y$

d) $\frac{m}{6} = 3$

$6 \times \frac{m}{6} = 6(3)$

$m = 18$

4. Solve each equation. Verify the solution.

a) $3q - 1 = 17$

$3q - 1 + 1 = 17 + 1$

$3q = 18$

$\frac{3q}{3} = \frac{18}{3}$

$q = 6$

$3(6) - 1 = 17$; the solution is correct.

b) $2(3 + p) = -4$

$2(3) + 2(p) = -4$

$6 + 2p = -4$

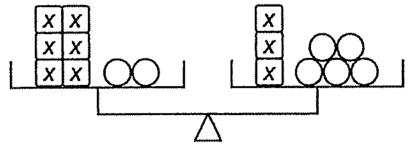
$6 + 2p - 6 = -4 - 6$

$2p = -10$

$p = -5$

$2(3 - 5) = -4$; the solution is correct.

6.2 **5.** Write the equation represented by the picture. Solve, and record your steps algebraically.



__6__ x + __2__ = __3__ x + __5__

$6x + 2 - 2 = 3x + 5 - 2$

$6x = 3x + 3$

$6x - 3x = 3x + 3 - 3x$

$3x = 3$

$x = 1$

6. Solve each equation. Verify the solution.

a) $3a - 2 = a - 6$

$3a - 2 + 2 = a - 6 + 2$

$3a = a - 4$

$3a - a = a - 4 - a$

$2a = -4$

$\frac{2a}{2} = \frac{-4}{2}$

$a = -2$

Left side $= 3a - 2$

$= 3(-2) - 2$

$= -8$

Right side $= a - 6$

$= -2 - 6$

$= -8$

$a =$ __-2__ is correct.

b) $4 + h = 1 - 2h$

$4 + h - 4 = 1 - 2h - 4$

$h = -3 - 2h$

$h + 2h = -3 - 2h + 2h$

$3h = -3$

$\frac{3h}{3} = \frac{-3}{3}$

$h = -1$

Left side $= 4 + h$

$= 4 + (-1)$

$= 3$

Right side $= 1 - 2h$

$= 1 - 2(-1)$

$= 3$

$h =$ __-1__ is correct.

c) $\frac{5a}{6} = 10$

$6 \times \frac{5a}{6} = 6(10)$

$5a = 60$

$\frac{5a}{5} = \frac{60}{5}$

$a = 12$

Left side $= \frac{5a}{6}$

$= \frac{5(12)}{6}$

$= 10$

Right side = __10__

$a =$ __12__ is correct.

6.3 Introduction to Linear Inequalities

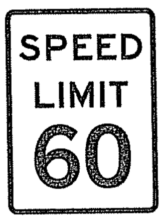
FOCUS Write and graph inequalities.

Less than	$<$	below, under
Less than or equal to	$\leq$	up to, at most, no more than, maximum
Greater than	$>$	over, more than
Greater than or equal to	$\geq$	at least, minimum

TEACHER NOTE
Investigate on Student Text page 288 is well suited to visual learners. Consider using mixed-ability groupings so that all students can participate.

Example 1 Writing an Inequality to Describe a Situation

Define a variable and write an inequality to describe the situation.

a) 

b) You must be at least 16 years old to get a driver's licence.

Solution

a) Let s represent the speed.
You can go up to 60 km/h, but not faster.
So, s can equal 60 or be any number less than 60.

The inequality is $s \leq 60$.

b) Let a represent the age to get a driver's licence.
"At least 16" means that you must be 16, or older.
You cannot be less than 16.
So, a can equal 16 or be greater than 16.

The inequality is $a \geq 16$.

$a \geq 16$ is read as a is ***greater than or equal to*** *16.*

Check

1. Let t represent the temperature in degrees Celsius.
Write an inequality to describe each situation:

a) For temperatures less than 0°C, make sure to wear warm clothing. t $\underline{\ <\ }$ 0

b) The highest temperature we've had this week was 12°C. t $\underline{\ \leq\ }$ 12

Linear Inequalities
A linear inequality may be true for many values of the variable.

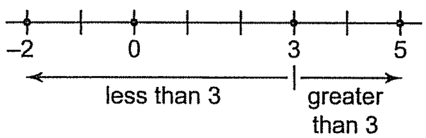
Example 2 Determining Whether a Number Is a Solution of an Inequality

Is each number a solution of the inequality $x \leq 3$? Justify the answers.

a) 5 **b)** 3 **c)** 0 **d)** −2

Solution

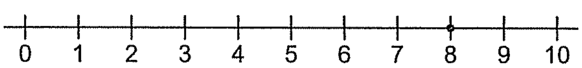
Use a number line to show all the numbers.
The solution of $x \leq 3$ is all numbers that are less than or equal to 3.



For a number to be less than 3, it must lie to the left of 3.
0 and −2 are to the left of 3, so they are solutions.
3 is equal to itself, so it is a solution.
5 is to the right of 3, so it is not a solution.

Check

1. a) Is 8 a solution of the inequality $x > 0$? Use the number line to help.

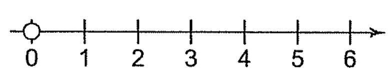


$x > 0$ is read as x is greater than 0.

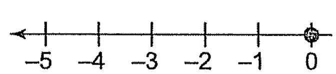
8 is to the **right** of 0, so 8 **is** a solution.

b) What are 3 other numbers that are solutions of $x > 0$?
Sample answers: 1, 2, 3

The solutions of an inequality can be graphed on a number line.
For example:
$a > 0$
a is greater than 0, so 0 is not included in the solution.
This is shown by an open circle at 0.



$z \leq 0$
z is less than or equal to 0, so 0 is included in the solution.
This is shown by a shaded circle at 0.



Example 3 Graphing Inequalities on a Number Line

Graph each inequality on a number line.
Write 3 numbers that are possible solutions of the inequality.

a) $b > 5$ **b)** $y \leq -1$ **c)** $-4 \geq n$ **d)** $-1 < r$

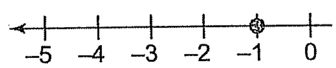
Solution

a) $b > 5$
Any number greater than 5 satisfies the inequality.

Draw an open circle at 5, because 5 is not part of the solution.

3 possible solutions are: 6, 7, 8

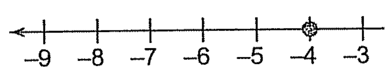
b) $y \leq -1$
Any number less than or equal to -1 satisfies the inequality.



Draw a shaded circle at -1, because -1 is part of the solution.

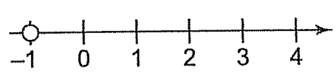
3 possible solutions are: -1, -2, -5

c) $-4 \geq n$ means -4 is greater than or equal to n, or n is less than or equal to -4.
$-4 \geq n$ is the same as $n \leq -4$.



3 possible solutions are: -4, -5, -6

d) $-1 < r$ means -1 is less than r, or r is greater than -1.
$-1 < r$ is the same as $r > -1$.

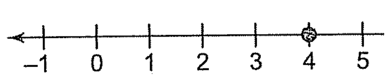


3 possible solutions are: 0, 2, 4

Check

1. Graph each inequality on a number line.
Write 3 numbers that are possible solutions for each inequality. **Sample answers**

a) $h \leq 4$



0, **1**, **4**

b) $-3 < x$

–2, **0**, **1**

Practice

1. Is each inequality true or false?
If it is false, change the sign to write a true inequality.

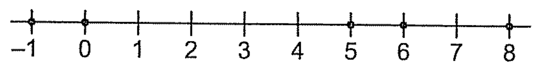
a) $3 < 10$ **True**

b) $3 < -10$ **False; $3 > -10$**

c) $0 \leq 1$ **True**

d) $1 \geq 1$ **True**

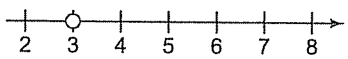
2. Is each number a solution of $x \geq 5$?



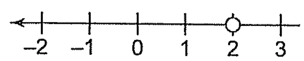
a) 5 **Yes** **b)** −1 **No** **c)** 0 **No** **d)** 8 **Yes** **e)** 6 **Yes**

3. a) Graph each inequality on the number line.

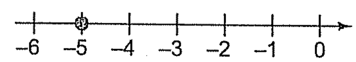
i) $m > 3$



ii) $x < 2$



iii) $y \geq -5$



b) Write 3 numbers that are possible solutions of each inequality above. **Sample answers**

i) **4, 5, 6** **ii)** **1, 0, −1** **iii)** **−5, −4, 0**

4. Write an inequality to model each situation.

a) The maximum speed is 100 km/h.
Let s represent the speed, in km/h. **$s \leq 100$**

b) The elevator can hold no more than 12 people. Let n represent the number of people the elevator can hold. **$n \leq 12$**

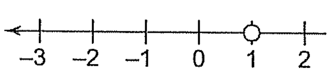
c) This year, the price of gas has always been at least 70 cents per litre. Let p represent the price of gas, in cents. **$p \geq 70$**

d) This pass card is good for up to 10 entries to the amusement park. Let n represent the number of entries. **$n \leq 10$**

5. Match each inequality with the graph of its solution below.

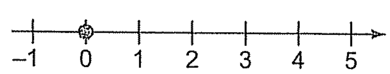
a) $x > 1$ **b)** $x \leq -2$ **c)** $x < 1$ **d)** $x \geq 0$

i)



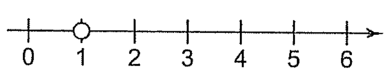
$x < 1$

ii)



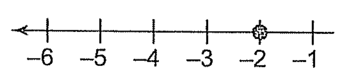
$x \geq 0$

iii)



$x > 1$

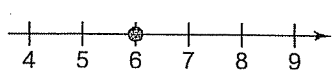
iv)



$x \leq -2$

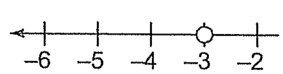
6. Write an inequality whose solution is graphed on the number line.

a)



$x \geq 6$

b)



$x < -3$

TEACHER NOTE
Next Steps:
Direct students to questions 3, 4, 5, 6, 7, 8, 9, and 12 on pages 292 and 293 of the Student Text.

6.4 Solving Linear Inequalities by Using Addition and Subtraction

FOCUS **Use addition and subtraction to solve inequalities.**

Consider the inequality $-1 < 3$.

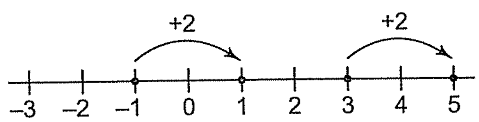
What happens to an inequality if we add the same number to each side?

$-1 < 3$ Add 2 to each side.

Left side: $-1 + 2 = 1$

Right side: $3 + 2 = 5$

The resulting inequality is still true: $1 < 5$



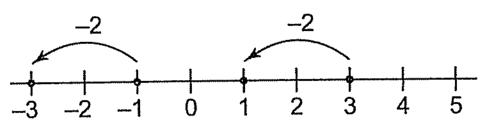
What happens to an inequality if we subtract the same number from each side?

$-1 < 3$ Subtract 2 from each side.

Left side: $-1 - 2 = -3$

Right side: $3 - 2 = 1$

The resulting inequality is still true: $-3 < 1$



Property of Inequalities

When the same number is added to or subtracted from each side of an inequality, the resulting inequality is still true.

The strategy that we used to solve an equation can be used to solve an inequality. Isolate the variable to solve.

Equation

$$r - 6 = -2$$
$$r - 6 + 6 = -2 + 6$$
$$r = 4$$

There is only 1 solution: $r = 4$

Inequality

$$r - 6 < -2$$
$$r - 6 + 6 < -2 + 6$$
$$r < 4$$

Any number less than 4 is part of the solution. The solution includes 3, 2, and 1, for example.

Example 1 Solving an Inequality

a) Solve the inequality $6 \le x - 4$.

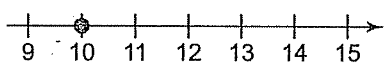
b) Graph the solution on a number line.

Solution

a)
$$6 \le x - 4$$
$$6 + 4 \le x - 4 + 4$$
$$10 \le x$$

Add 4 to each side to isolate x.

This is the same as $x \ge 10$.

b) 

Place a shaded circle on 10 because 10 is part of the solution.

Check

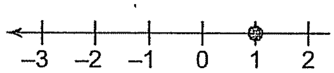
1. Solve each inequality. Graph the solution on the number line.

a) $p + 3 \le 4$

$p + 3 - 3 \le 4 - 3$

$p \le 4 - 3$

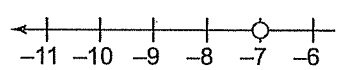
$p \le 1$



b) $-5 > 2 + a$

$-5 - 2 > 2 + a - 2$

$-7 > a$, or $a < -7$



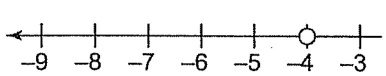
Example 2 Solving an Inequality with Variables on Both Sides

a) Solve the inequality $3d + 2 < 2d - 2$.

b) Graph the solution on a number line.

Solution

a)
$$3d + 2 < 2d - 2$$
Subtract $2d$ from each side.
$$3d + 2 - 2d < 2d - 2 - 2d$$
$$d + 2 < -2$$
Subtract 2 from each side.
$$d + 2 - 2 < -2 - 2$$
$$d < -4$$

b) 

Place an open circle on −4 because −4 is not part of the solution.

Check

1. Solve each inequality. Graph the solution on a number line.

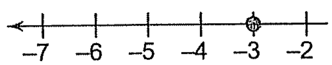
a) $3z + 1 \leq 2z - 2$

$3z + 1 - 1 \leq 2z - 2 - 1$

$3z \leq 2z - 3$

$3z - 2z \leq 2z - 3 - 2z$

$z \leq -3$



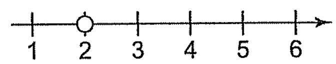
b) $4 - 4x > 6 - 5x$

$4 - 4x - 4 > 6 - 5x - 4$

$-4x > 2 - 5x$

$-4x + 5x > 2 - 5x + 5x$

$x > 2$



Practice

1. Which operation will you perform to each side of the inequality to isolate the variable?

a) $a + 1 > 3$
Subtract 1

b) $2 < m - 3$
Add 3

c) $x - 4 \geq 5$
Add 4

d) $6 > 1 - z$
Subtract 1

2. Fill in the missing steps to get to the solution.

a) $x + 5 > 10$

$x + 5 - 5 > 10 - 5$

$x > 5$

b) $12 \leq x - 4$

$12 + 4 \leq x - 4 + 4$

$16 \leq x$

3. Solve each inequality.
Match each inequality with the graph of its solution, below.

a) $n - 4 > -2$

$n - 4 + 4 > -2 + 4$

$n > 2$

b) $p + 6 < -2$

$p + 6 - 6 < -2 - 6$

$p < -8$

c) $u - 3 \geq -4$

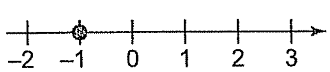
$u - 3 + 3 \geq -4 + 3$

$u \geq -1$

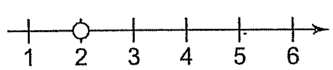
d) $2 + y > -2$

$2 + y - 2 > -2 - 2$

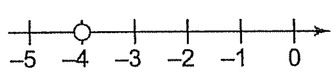
$y > -4$

i) 

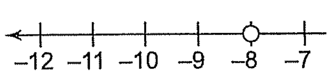
$u \geq -1$

ii) 

$n > 2$

iii) 

$y > -4$

iv) 

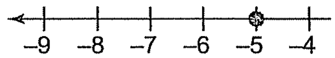
$p < -8$

4. a) Solve each inequality. Graph the solution on a number line.

i) $y + 3 \leq -2$

$\mathbf{y + 3 - 3 \leq -2 - 3}$

$\mathbf{y \leq -5}$



ii) $2 + b < 5$

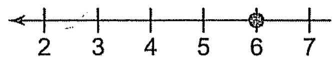
$\mathbf{2 + b - 2 < 5 - 2}$

$\mathbf{b < 3}$

iii) $4 \geq n - 2$

$\mathbf{4 + 2 \geq n - 2 + 2}$

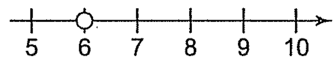
$\mathbf{6 \geq n}$**, or** $\mathbf{n \leq 6}$



iv) $3 < t - 3$

$\mathbf{3 + 3 < t - 3 + 3}$

$\mathbf{6 < t}$**, or** $\mathbf{t > 6}$



b) Write 3 numbers that are possible solutions for each inequality. **Sample answers**

i) **−5, −6, −7**

ii) **0, 1, 2**

iii) **4, 5, 6**

iv) **8, 9, 10**

c) Write a number that is NOT a solution of each inequality. **Sample answers**

i) **−4**

ii) **3**

iii) **7**

iv) **5**

TEACHER NOTE

Next Steps: Direct students to questions 4, 5, 7, 8, 9 on page 298 of the Student Text.

For students experiencing success, introduce Example 2 on page 297 of the Student Text. Assign Practice questions 12 and 13.

5. Solve, then graph each inequality.

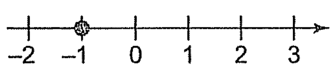
a) $6a + 2 \geq 5a + 1$

$\mathbf{6a + 2 - 2 \geq 5a + 1 - 2}$

$\mathbf{6a \geq 5a - 1}$

$\mathbf{6a - 5a \geq 5a - 1 - 5a}$

$\mathbf{a \geq -1}$



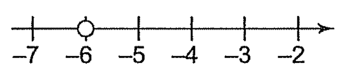
b) $3 + 2s > s - 3$

$\mathbf{3 + 2s - 3 > s - 3 - 3}$

$\mathbf{2s > s - 6}$

$\mathbf{2s - s > s - 6 - s}$

$\mathbf{s > -6}$



6. a) Solve the equation: $4v - 6 = 3v - 3$

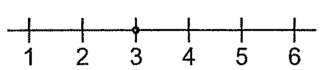
$\mathbf{4v - 6 + 6 = 3v - 3 + 6}$

$\mathbf{4v = 3v + 3}$

$\mathbf{4v - 3v = 3v + 3 - 3v}$

$\mathbf{v = 3}$

Graph the solution.



b) Solve the inequality: $4v - 6 > 3v - 3$

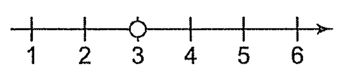
$\mathbf{4v - 6 + 6 > 3v - 3 + 6}$

$\mathbf{4v > 3v + 3}$

$\mathbf{4v - 3v > 3v + 3 - 3v}$

$\mathbf{v > 3}$

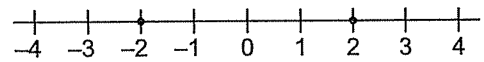
Graph the solution.



6.5 Solving Linear Inequalities by Using Multiplication and Division

FOCUS **Use multiplication and division to solve inequalities.**

Consider the inequality $-2 < 2$.



What happens to an inequality when we multiply or divide each side by the same positive number?

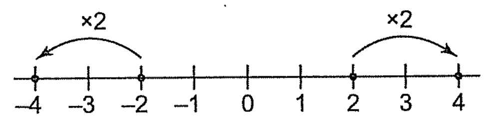
$-2 < 2$ Multiply each side by 2.

Left side: $(-2)(2) = -4$

Right side: $2(2) = 4$

$-4 < 4$

The resulting inequality is still true.



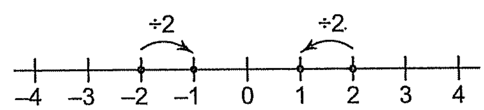
$-2 < 2$ Divide each side by 2.

Left side: $\frac{-2}{2} = -1$

Right side: $\frac{2}{2} = 1$

$-1 < 1$

The resulting inequality is still true.



> **Property of Inequalities**
> When each side of an inequality is multiplied or divided by the same positive number, the resulting inequality is still true.

What happens to an inequality when we multiply or divide each side by the same negative number?

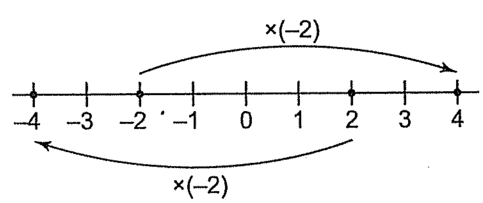
$-2 < 2$ Multiply each side by -2.

Left side: $(-2)(-2) = 4$

Right side: $2(-2) = -4$

$4 > -4$

For the inequality to be true, the sign has to be reversed.



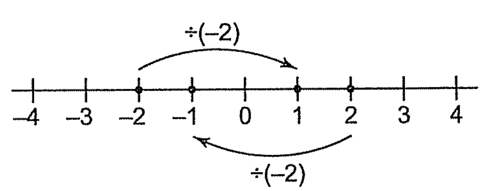
$-2 < 2$ Divide each side by -2.

Left side: $\frac{-2}{-2} = 1$

Right side: $\frac{2}{-2} = -1$

$1 > -1$

For the inequality to be true, the sign has to be reversed.



Property of Inequalities
When each side of an inequality is multiplied or divided by the same negative number, the inequality sign must be reversed for the inequality to remain true.

Example 1 Solving One-Step Inequalities

Solve each inequality and graph the solution.

a) $4x < -12$ **b)** $-2c \geq 8$ **c)** $\frac{b}{2} \leq 3$ **d)** $\frac{v}{-3} > 4$

Solution

a) $4x < -12$ Divide each side by 4.

$\frac{4x}{4} < \frac{-12}{4}$

When you divide each side by the same positive number, do not reverse the inequality sign.

$x < -3$

The solution of $x < -3$ is all numbers less than -3.

b) $-2c \geq 8$ Divide each side by -2.

$\frac{-2c}{-2} \leq \frac{8}{-2}$

When you divide each side by the same negative number, reverse the inequality sign.

$c \leq -4$

The solution of $c \leq -4$ is all numbers less than or equal to -4.

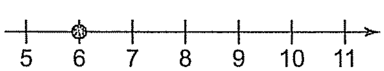
c) $\frac{b}{2} \geq 3$ Multiply each side by 2.

$2 \times \frac{b}{2} \geq 2(3)$

When you multiply each side by the same positive number, do not reverse the inequality sign.

$b \geq 6$

The solution of $b \geq 6$ is all numbers greater than or equal to 6.



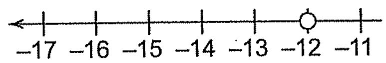
d) $\frac{v}{-3} > 4$ Multiply each side by -3.

$(-3)\left(\frac{v}{-3}\right) < (-3)(4)$

When you multiply each side by the same negative number, reverse the inequality sign.

$v < -12$

The solution of $v < -12$ is all numbers less than -12.



Check

1. State whether you would reverse the inequality sign to solve each inequality.

a) $-2m < 8$
Reverse

b) $2m \leq 8$
Do not reverse

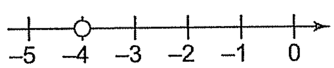
c) $\frac{y}{-2} > 3$
Reverse

2. Solve the inequalities in question 1. Graph each solution.

a) $-2m < 8$

$\frac{-2m}{-2} > \frac{8}{-2}$

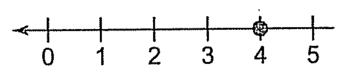
$m > -4$



b) $2m \leq 8$

$\frac{2m}{2} \leq \frac{8}{2}$

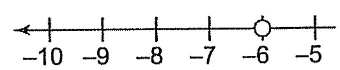
$m \leq 4$



c) $\frac{y}{-2} > 3$

$(-2)\left(\frac{y}{-2}\right) < (-2)3$

$y < -6$



Example 2 Solving a Multi-Step Inequality

a) Solve the inequality: $1 - \frac{2}{3}x > 3$

b) Graph the solution.

Solution

a) $1 - \frac{2}{3}x > 3$ Subtract 1 from each side to isolate x.

$1 - \frac{2}{3}x - 1 > 3 - 1$

$-\frac{2}{3}x > 2$ Multiply each side by –3 to clear the fraction. Reverse the inequality sign.

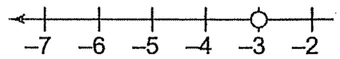
$(-3)\left(-\frac{2}{3}x\right) < (-3)(2)$

$2x < -6$ Divide each side by 2.

$\frac{2x}{2} < \frac{-6}{2}$

$x < -3$

b) The solution of $x < -3$ is all numbers less than -3.



Check

1. Solve the inequality: $-\frac{2f}{5} < 4$

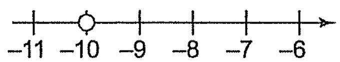
Graph the solution on the number line.

$\mathbf{5\left(\frac{-2f}{5}\right) < 5(4)}$

$\mathbf{-2f < 20}$

$\mathbf{\frac{-2f}{-2} > \frac{20}{-2}}$

$\mathbf{f > -10}$



If you multiply or divide by a negative number, remember to reverse the inequality sign.

Practice

1. a) Will the inequality sign change when you perform the indicated operation on each side of the inequality?

i) $3 > -2$; Multiply by 2 — **No**

ii) $4 \leq 8$; Divide by -4 — **Yes**

iii) $-5 < 1$; Multiply by -5 — **Yes**

iv) $1 > -4$; Divide by 1 — **No**

b) Perform each operation above. Write the resulting inequality.

i) $\mathbf{6 > -4}$ **ii)** $\mathbf{-1 \geq -2}$

iii) $\mathbf{25 > -5}$ **iv)** $\mathbf{1 > -4}$

2. a) For the inequality $-2 < 6$, identify which of the following operations will reverse the inequality sign.

i) Multiply both sides by -4 — **Yes**

ii) Divide both sides by 2 — **No**

b) Perform each operation above. Write the resulting inequality.

i) $\mathbf{8 > -24}$ **ii)** $\mathbf{-1 < 3}$

3. a) What operation do you have to do to solve each inequality?

i) $3x > 9$ — **Divide by 3**

ii) $-4p < -8$ — **Divide by -4**

iii) $-3y \leq 15$ — **Divide by -3**

iv) $\frac{q}{-2} \leq 5$ — **Multiply by -2**

b) State whether you would reverse the inequality sign to solve each inequality in part a.

i) **No** **ii)** **Yes**

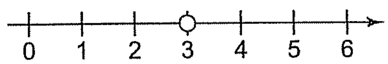
iii) **Yes** **iv)** **Yes**

c) Solve and graph each inequality.

i) $3x > 9$

$\frac{3x}{3} > \frac{9}{3}$

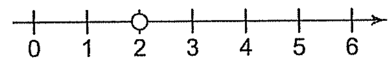
$x > 3$



ii) $-4p < -8$

$\frac{-4p}{-4} > \frac{-8}{-4}$

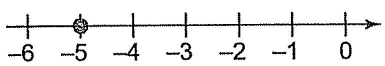
$p > 2$



iii) $-3y \leq 15$

$\frac{-3y}{-3} \geq \frac{15}{-3}$

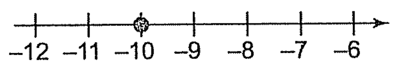
$y \geq -5$



iv) $\frac{q}{-2} \leq 5$

$(-2)\left(\frac{q}{-2}\right) \geq (-2)(5)$

$q \geq -10$



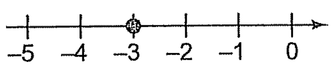
4. Solve each inequality. Graph the solution.

a) $3 - 2r \leq 9$

$3 - 2r - 3 \leq 9 - 3$

$-2r \leq 6$

$r \geq -3$

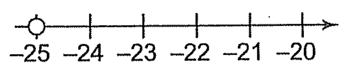


b) $\frac{p}{5} + 2 > -3$

$\frac{p}{5} + 2 - 2 > -3 - 2$

$\frac{p}{5} > -5$

$p > -25$

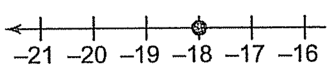


c) $\frac{-s}{6} \geq 3$

$6\left(\frac{-s}{6}\right) \geq 6(3)$

$-s \geq 18$

$s \leq -18$



d) $\frac{5w}{8} - 1 < 4$

$\frac{5w}{8} - 1 + 1 < 4 + 1$

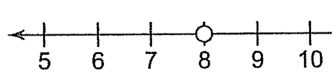
$\frac{5w}{8} < 5$

$8 \times \frac{5w}{8} < 5(8)$

$5w < 40$

$\frac{5w}{5} < \frac{40}{5}$

$w < 8$



TEACHER NOTE

Next Steps: Direct students to questions 3, 5a, 6, 7, 9a, b, and 11a, b on page 305 of the Student Text.

For students experiencing success, introduce Example 3 on page 304 of the Student Text. Assign Practice questions 8 and 10.

Unit 6 Puzzle

How Great Is My Number?

You will need

10 red tiles, 10 yellow tiles, 2 die

Label 1 die with the following faces:

>	<	=	≥	≤	=

Number of Players

2

Goal of the Game

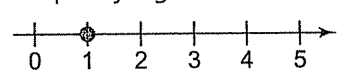
To get 4 tiles in a row, vertically, horizontally, or diagonally

8	−1	0	2	−2
−4	9	−5	4	1
3	7	6	5	−9
−12	−10	−7	−3	11
−6	−8	10	12	−11

How to Play

1. Roll the number die.
   The player with the greater number goes first.
2. The starting player rolls the die, and covers a number on the board that corresponds to what was rolled. For example, if a player rolls ≥ •, the player can cover any number that is greater than or equal to 1.
3. Only 1 number can be covered on a turn.
4. Players alternate turns.
5. The first player to get 4 tiles in a row wins.

Unit 6 Study Guide

Skill	Description	Example
Solving Equations	To solve an equation, find the value of the variable that makes the left side of the equation equal to the right side. To solve an equation, isolate the variable on one side of the equation. Use inverse operations or a balance strategy to perform the same operation on both sides of the equation: • Add the same quantity to each side • Subtract the same quantity from each side • Multiply or divide each side by the same non-zero quantity Algebra tiles and balance scales can help model the steps in the solution.	Solve the equation: $3y - 2 = y + 4$ **Solution** $3y - 2 = y + 4$ $3y - 2 + 2 = y + 4 + 2$ $3y = y + 6$ $3y - y = y - y + 6$ $2y = 6$ $\frac{2y}{2} = \frac{6}{2}$ $y = 3$
Solving Inequalities	An inequality is a statement that one quantity is less than (<) another, greater than (>) another, less than or equal to (≤) another, or greater than or equal to (≥) another. The inequality sign reverses when you multiply or divide each side of the inequality by the same negative number. A linear inequality may be true for many values of the variable. We can graph the solutions on a number line.	Solve the inequality and graph the solution: $-2s - 2 \leq s - 5$ **Solution** $-2s - 2 + 2 \leq s - 5 + 2$ $-2s \leq s - 3$ $-2s - s \leq s - 3 - s$ $-3s \leq -3$ $\frac{-3s}{-3} \geq \frac{-3}{-3}$ $s \geq 1$ Since we divide each side by the same negative number, the inequality sign is reversed. 

Unit 6 Review

6.1 **1.** Solve each equation. Verify the results.

a) $f + 6 = 3$

$f + 6 - 6 = 3 - 6$

$f = -3$

Check: $-3 + 6 = 3$

$f =$ -3 is correct.

b) $g - 5 = -2$

$g - 5 + 5 = -2 + 5$

$g = 3$

Check: $3 - 5 = -2$

$g =$ 3 is correct.

c) $5h = 25$

$\frac{5h}{5} = \frac{25}{5}$

$h = 5$

Check: $5(5) = 25$

$h =$ 5 is correct.

d) $-2k = 6$

$\frac{-2k}{-2} = \frac{6}{-2}$

$k = -3$

Check: $-2(-3) = 6$

$k =$ -3 is correct.

2. Solve each equation. Verify the solution.

a) $4x - 2 = 6$

$4x - 2 + 2 = 6 + 2$

$4x = 8$

$\frac{4x}{4} = \frac{8}{4}$

$x = 2$

Left side $= 4x - 2$

$= 4(2) - 2$

$= 6$

Right side $= 6$

$x =$ 2 is correct.

b) $2 - 3c = -7$

$2 - 3c - 2 = -7 - 2$

$-3c = -9$

$\frac{-3c}{-3} = \frac{-9}{-3}$

$c = 3$

Left side $= 2 - 3c$

$= 2 - 3(3)$

$= -7$

Right side $= -7$

$c =$ 3 is correct.

c) $2v - 3 = -9$

$2v - 3 + 3 = -9 + 3$

$2v = -6$

$\frac{2v}{2} = \frac{-6}{2}$

$v = -3$

Left side $= 2v - 3$

$= 2(-3) - 3$

$= -9$

Right side $= -9$

$v =$ -3 is correct.

d) $-2(2 + w) = -20$

$(-2)(2) + (-2)(w) = -20$

$-4 - 2w = -20$

$-4 - 2w + 4 = -20 + 4$

$-2w = -16$

$\frac{-2w}{-2} = \frac{-16}{-2}$

$w = 8$

Left side $= -2(2 + w)$

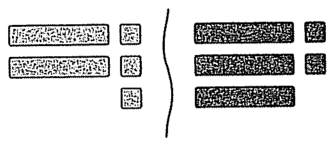
$= -2(2 + 8)$

$= -20$

Right side $= -20$

$w =$ 8 is correct.

6.2 **3.** Write the equation modelled by each set of algebra tiles. Solve the equation.

a) 

$2a + 3 = -3a - 2$

$2a + 3a + 3 = -3a - 2 + 3a$

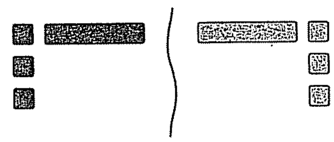
$5a + 3 = -2$

$5a + 3 - 3 = -2 - 3$

$5a = -5$

$\frac{5a}{5} = \frac{-5}{5}$

$a = -1$

b) 

$-3 - x = x + 3$

$-3 - x + 3 = x + 3 + 3$

$-x = x + 6$

$-x - x = x + 6 - x$

$-2x = 6$

$\frac{-2x}{-2} = \frac{6}{-2}$

$x = -3$

4. Solve each equation.

a) $9 - 2w = w - 6$

$9 - 2w - w = w - 6 - w$

$9 - 3w = -6$

$9 - 3w - 9 = -6 - 9$

$-3w = -15$

$\frac{-3w}{-3} = \frac{-15}{-3}$

$w = 5$

b) $e - 6 = 6 - e$

$e - 6 + e = 6 - e + e$

$2e - 6 = 6$

$2e - 6 + 6 = 6 + 6$

$2e = 12$

$\frac{2e}{2} = \frac{12}{2}$

$e = 6$

c) $3n + 1 = 3 + n$

$3n + 1 - n = 3 + n - n$

$2n + 1 = 3$

$2n + 1 - 1 = 3 - 1$

$2n = 2$

$\frac{2n}{2} = \frac{2}{2}$

$n = 1$

d) $m - 2 = 3m + 4$

$m - 2 - 3m = 3m + 4 - 3m$

$-2m - 2 = 4$

$-2m - 2 + 2 = 4 + 2$

$-2m = 6$

$\frac{-2m}{-2} = \frac{6}{-2}$

$m = -3$

5. Solve each equation. Verify the solution.

a) $6 + \frac{s}{2} = 7$

$6 + \frac{s}{2} - 6 = 7 - 6$

$\frac{s}{2} = 1$

$2 \times \frac{s}{2} = 2(1)$

$s = 2$

Left side $= 6 + \frac{s}{2}$

$= 6 + \frac{2}{2}$

$= 7$

Right side = 7

$s =$ 2 is correct.

b) $4 + \frac{2x}{3} = 2$

$\mathbf{4 + \frac{2x}{3} - 4 = 2 - 4}$

$\mathbf{\frac{2x}{3} = -2}$

$\mathbf{3 \times \frac{2x}{3} = 3(-2)}$

$\mathbf{2x = -6}$

$\mathbf{\frac{2x}{2} = \frac{-6}{2}}$

$\mathbf{x = -3}$

Left side $= 4 + \frac{2x}{3}$

$\mathbf{= 4 + \frac{2(-3)}{3}}$

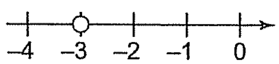
$\mathbf{= 4 - 2}$

$\mathbf{= 2}$

Right side $=$ **2**

$x =$ **−3** is correct.

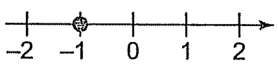
6.3 **6.** Graph each inequality.
Write 3 numbers that are possible solutions for each inequality. **Sample answers**

a) $q > -3$ 

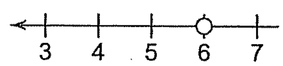
0, 1, 2

b) $w \leq 0$ 

0, −1, −2

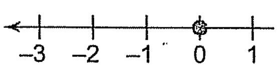
c) $t \geq -1$ 

−1, 0, 1

d) $r < 6$ 

5, 4, 3

7. Write an inequality whose solution is graphed on the number line.

a) 

$\mathbf{a \leq 0}$

b) 

$\mathbf{s > 2}$

6.4 **8.** Solve each inequality. Graph the solution.

a) $d - 6 > 4$

$\mathbf{d - 6 + 6 > 4 + 6}$

$\mathbf{d > 10}$

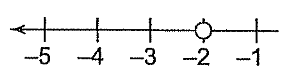
b) $2f + 1 < -3$

$\mathbf{2f + 1 - 1 < -3 - 1}$

$\mathbf{2f < -4}$

$\mathbf{\frac{2f}{2} < \frac{-4}{2}}$

$\mathbf{f < -2}$



9. Solve each inequality. Graph the solution.

a) $4j - 1 \geq 2j + 3$

$\mathbf{4j - 1 + 1 \geq 2j + 3 + 1}$

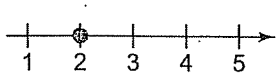
$\mathbf{4j \geq 2j + 4}$

$\mathbf{4j - 2j \geq 2j + 4 - 2j}$

$\mathbf{2j \geq 4}$

$\mathbf{\frac{2j}{2} \geq \frac{4}{2}}$

$\mathbf{j \geq 2}$



b) $k - 2 < 2 - k$

$\mathbf{k - 2 + 2 < 2 - k + 2}$

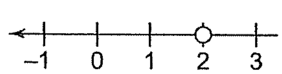
$\mathbf{k < 4 - k}$

$\mathbf{k + k < 4 - k + k}$

$\mathbf{2k < 4}$

$\mathbf{\frac{2k}{2} < \frac{4}{2}}$

$\mathbf{k < 2}$



6.5 **10.** State whether you would reverse the inequality sign to solve each inequality.

a) $2z < -4$ **Do not reverse**

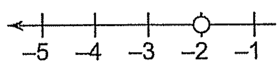
b) $-2x \geq 4$ **Reverse**

c) $\frac{c}{-2} < 4$ **Reverse**

d) $\frac{v}{2} \geq -4$ **Do not reverse**

11. Solve each inequality in question 10. Graph the solution.

a) $2z < -4$



$\frac{2z}{2} < \frac{-4}{2}$

$z < -2$

b) $-2x \geq 4$

$\frac{-2x}{-2} \leq \frac{4}{-2}$

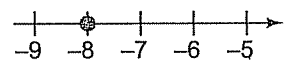
$x \leq -2$

c) $\frac{c}{-2} < 4$

$(-2)\left(\frac{c}{-2}\right) > (-2)(4)$

$c > -8$

d) $\frac{v}{2} \geq -4$



$2 \times \frac{v}{2} \geq 2(-4)$

$v \geq -8$

12. Solve each inequality and graph the solution.

a) $-3b + 4 \geq -5$

$-3b + 4 - 4 \geq -5 - 4$

$-3b \geq -9$

$\frac{-3b}{-3} \leq \frac{-9}{-3}$

$b \leq 3$

b) $n + 2 < 2n - 2$

$n + 2 - 2 < 2n - 2 - 2$

$n < 2n - 4$

$n - 2n < 2n - 4 - 2n$

$-n < -4$

$n > 4$

c) $-5 - m < 3 + m$

$-5 - m - 3 < 3 + m - 3$

$-8 - m < m$

$-8 - m + m < m + m$

$-8 < 2m$

$\frac{-8}{2} < \frac{2m}{2}$

$-4 < m$

d) $2 - \frac{x}{2} > 1$

$2 - \frac{x}{2} - 2 > 1 - 2$

$-\frac{x}{2} > -1$

$(-2)\left(-\frac{x}{2}\right) < (-2)(-1)$

$x < 2$

UNIT 7

Similarity and Transformations

What You'll Learn

- Draw and interpret scale diagrams.
- Apply properties of similar polygons.
- Identify and describe line symmetry and rotational symmetry.

Why It's Important

Similarity and scale diagrams are used by

- construction workers when they construct buildings and bridges
- motorists when they use maps to get around a city

Symmetry is used by

- interior designers when they arrange furniture and accessories in a room

Key Words

enlargement
reduction
scale diagram
scale factor
polygon
non-polygon
similar polygons
proportional
line symmetry
congruent
reflection
line of reflection
tessellation
rotation
rotational symmetry
order of rotation
angle of rotation symmetry
translation